MATHEMATICS WITHOUT BORDERS

WINTER 2019

AGE GROUP 9

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Simplify the following expression $\left||1-\sqrt{2}|+|\sqrt{2}-\sqrt{3}|\right|+1$.

Problem 2. If $x + y + z = 3$ and $x^2+y^2+z^2=9,$ calculate $xy+yz+zx$.

Problem 3. The numbers a, b and c are different and

$$\frac{b+c}{a}=\frac{c+a}{b}=\frac{a+b}{c}.$$

Find the following sum:

$$\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}.$$

Problem 4. If the number β is such that $\beta^2+5\beta+1=0$, calculate the value of the following expression:

$$\beta^2+\frac{1}{\beta^2}.$$

Problem 5. Find the least natural number that has a product of its digits equal to $6^7\times 10^2$.

Problem 6. The number a_1 is an integer, and

$$a_2=a_1+1, a_3=a_2+1, a_4=a_3+1, a_5=a_4+1, a_6=a_5+1, a_7= \\ =a_6+1, a_8=a_7+1, a_9=a_8+1.$$

If $a_1+a_2+a_3=a_4+a_5+a_6+a_7+a_8+a_9,$ calculate

$a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9$.

Problem 7. In how many parts can a plane be divided by three straight lines?

Problem 8. If a, b and c are integers; $A=(a+b)\times(b+c)\times(c+a)$; and A is divisible by 3, find the number of possible remainders left after dividing A by 6.

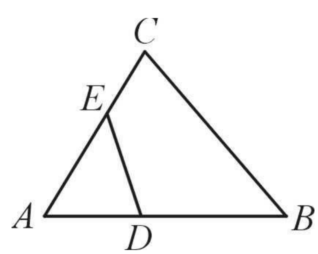
Problem 9. Find the least value of the natural number n, for which $3^n + 2$ is a composite number?

Problem 10. The product of the rational number *R* and the irrational number *I* is a rational number. The sum of *R* and *I* is $\sqrt{3} + 2$. Calculate $R^2 + I^2$.

Problem 11. A square has been inscribed in a larger square with a side length of 4 cm. Find the smallest possible side length of the inscribed square in centimetres.

Problem 12. The points *M* (5, *m*) and *N* (*n*, *p*) belong to the parabola $y = x^2 - 4x + 3$. The point *M* is symmetric to the point *N* with respect to the straight line *x* = 2. Calculate $m + n + p$.

Problem 13. Let *AE* = 3 *cm*, *EC* = *5* cm, *AD* = 4 *cm*, *DB* = *2* cm, *ED* = 2 *cm*. Find the length of the side *BC* in cm.



Problem 14. In the isosceles triangle *ABC* (*AC* = *BC*), $\angle CAB = 70^0$. The point *D* is external to the triangle, and $\angle ADB = 20^0, \angle CDA = 30^0$.
How many degrees is $\angle CAD$?

Problem 15. The point *M* is the midpoint of the side *CD* in the square *ABCD* with a side length of $3\sqrt{2}$ *cm*. The straight lines *AC* and *BM* intersect at point *N*. Find the length of the line segment *AN* in cm.

Задача 16. We have 9 different flowers, which we need to use to make a bouquet of either 3, 5 or 7 flowers. How many different bouquets can we make?

Problem 17. How many solutions does the following system of inequalities have?

$$\begin{cases} 7x_1 \geq 2x_3^2 + 3x_3 + 2 \\ 7x_2 \geq 2x_1^2 + 3x_1 + 2 \\ 7x_3 \geq 2x_2^2 + 3x_2 + 2? \end{cases}$$

Problem 18. Simplify the following expression:

$$\sqrt{x^2 - 2x + 1} + \sqrt{x^2 - 4x + 4},$$

if $1 \leq x \leq 2$.

Problem 19. The numbers 1, 2, 3, ..., 8 and 9 were divided into three groups. Each group contains three numbers. If A is the greatest possible product of the numbers in one of the three groups, find the least value of A.

Problem 20. Let p and q be such that $4p + 4q + 1 < 0$. How many real roots does the following equation have: $(x^2 - 2px + q) \times (x^2 - 2qx + p) = 0$?