MATHEMATICS WITHOUT BORDERS

AGE GROUP 9

SPRING 2019

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Calculate the product of x, y and z, if

$$\begin{cases} 2^x = 3 \\ 3^y = 5 \\ 5^z = 0.25. \end{cases}$$

Problem 2. Two of the roots of the equation $ax^4 + bx^2 + c = 0$ (where a and b are real numbers) are the numbers 1 and 2. Calculate the sum of the other two roots.

Problem 3. Calculate $A - B$, if

$$\sqrt{A^2 + 2A + 2} + \sqrt{B^2 + 4B + 5} = 2.$$

Problem 4. The natural numbers from 1 to 25 have been written on cards. Some cards are picked without looking. At least how many cards should we pick to make sure that 2 of them will have a number divisible by 10 as their product?

Problem 5. Calculate the following expression:

$$\sqrt{(1-\sqrt{3})^2} \times \left(1+\sqrt{3}\right) + \sqrt{(1-\sqrt{2})^2} \times \left(1+\sqrt{2}\right).$$

Problem 6. For how many integers a are both roots of the quadratic equation

$$a^2x^2 + (a^2 - 4)x + a^2 = 0$$

positive numbers?

Problem 7. How many natural numbers n are there, for which exactly one of the numbers n and $n + 2019$ is a four-digit number?

Problem 8. Let $ABCD$ be a trapezium (AB and CD are parallel, $AB < CD$). The circumcircle of the triangle ABC touches AD. If $AC = 6$ cm and $AB = 4$ cm, calculate the length of the base CD in cm.

Problem 9. The triangle ABC ($AC = BC$) is isosceles. The point L lies on BC such that the line AL bisects $\measuredangle CAB$. If $AC + CL = AB$, calculate $\measuredangle CBA$.

Problem 10. Two of the sides of a triangle have lengths of $\sqrt{2}$ cm and $\sqrt{3}$ cm, respectively. The perpendicular height of one side is 2cm longer than the perpendicular height of the other side. Find the area of the triangle in cm^2.

Problem 11. The equilateral triangles ΔADM and ΔDCN have been built to the outside of the parallelogram $ABCD$ with $\measuredangle BAD = 30^0$ and an area of 10 cm^2. Find the area of ΔMDN in cm^2.

Problem 12. In a rectangular coordinate system, the vertices of the triangle ABC have the following coordinates: A(0;0), B(4;0), C (1;6). Find the coordinates of the centroid of the triangle.

Problem 13. If x_1 and x_2 are roots of the equation $x^2 - 7x + 3 = 0$, calculate

$$\frac{5x_1}{x_1^2 + 3} + \frac{2x_2}{x_2^2 + 3}.$$

Problem 14. Let *A, B* and *C* be integers, such that $(x - A)(x - 2) + 1 = (x + B)(x + C)$ is an identity. Calculate *B* + *C*.

Problem 15. Find the sum of the two-digit numbers $\overline{ab}$ and $\overline{ba}$, if

$$600 < (\overline{ab})^2 - (\overline{ba})^2 < 700?$$

Problem 16. For which primes *x* < 10 are there only 2 primes that are factors of

$$x^{2019} + 6x^{2018} + 9x^{2017}?$$

Problem 17. For which natural numbers *n* are $\frac{n+6}{n-1}$ and $\frac{3n+6}{2n-6}$ integers?

Problem 18. In how many ways can we separate 9 children into two groups? One of the groups must have 6 children, and the other must have 3.

Problem 19. Let *x*, *y* and *z* be natural numbers, such that

$$|x - y| + |y - 1| + |z - 2| = 3.$$

Calculate the greatest possible value of $x + y + z$.

Problem 20. Find the last digit of the difference of the following numbers:

$$2015^{2019} - 2016^{2020}.$$