MATHEMATICS WITHOUT BORDERS

WINTER 2019

AGE GROUP 8

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. If $|1-m|+1+n^2=2n$, calculate $n+m$.

Problem 2. The polynomial x^5+1 can be expressed as the product of two polynomials: $(x+1)$ and $(Ax^4+Bx^3+Cx^2+Dx+E)$.
Calculate $A - B + C - D + E$.

Problem 3. Calculate the sum of all positive two-digit numbers, which, when divided by 6, leave a remainder of 4.

Problem 4. The numbers *A* and *B* are negative integers, and $19=A^3-B^3$.
Calculate $A+9\times B$.

Problem 5. Find the smallest natural number with a product of its digits equal to $6^7\times 10^2$.

Problem 6. The number a_1 is an integer, and
$a_2=a_1+1, a_3=a_2+1, a_4=a_3+1, a_5=a_4+1, a_6=a_5+1, a_7=a_6+1, a_8=a_7+1, a_9=a_8+1.$
If $a_1+a_2+a_3=a_4+a_5+a_6+a_7+a_8+a_9$, how many of the numbers $a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8,\ a_9$ are positive?

Problem 7. There were exactly four Mondays and exactly four Fridays in a particular January. What day of the week did the 1st of January fall on during this year?

Problem 8. Let *a*, *b* and *c* be integers, and let $A=(a+b)\times(b+c)\times(c+a)$.
If *A* is divisible by 3, find the number of possible remainders left after dividing *A* by 6.

Problem 9. Find the least value of the natural number *n*, for which the number 3^n+2 is composite.

Problem 10. The product of the rational number *R* and the irrational number *I* is a rational number. The sum of *R* and *I* is $\sqrt{3}+1$. Calculate R^2+I^2.

Problem 11. The distance between the centres of two circles with radii of 1 cm and 2 cm is 4 cm. Calculate the greatest possible distance between two points, one of which lies on one of the circles, and the other point – on the other circle.

Problem 12. A wooden sphere has been cut into two identical parts. The surface area of one of the resulting solids is 60 cm^2. Find the surface area of the sphere.

Hint:

The surface area of a sphere with a radius R is $4\pi R^2$.

The area of a circle with a radius r is πr^2.

Problem 13. The square *ABCD* and the point *F* lie on the same plane.

If *CF* = *FD* = *AB*, calculate the greatest value of $\measuredangle AFB$ in degrees.

Problem 14. 3 blue points, 5 green points and *N* red points have been placed along a circle.

The segments with endpoints of identical colours are 37 less than the segments with endpoints of different colours. Find *N*.

Problem 15. Three metal cubes with edges of 3 cm, 4 cm and 5cm, respectively, have been melted in order to form a new cube. Find the volume of the new cube in cm^3.

Problem 16. The numbers 3644 and 3541 were both divided by the natural number *X* and the resulting remainders were the same. Find the number *X*.

Problem 17. Peter added the numbers which belong to the set B $\{-3, -5, 1, 2\}$, but belong neither to A $\{-2, -3, -6, 5\}$, nor to C $\{-1, -2, -3, -6, 2, 5\}$. John added the numbers which belong to A and C, but do not belong to B. By how much is John's sum greater than Peter's sum?

Problem 18. How many different triangles can we create by using 3 out of 5 segments with the following lengths: 1 *cm*, 2 *cm*, 3 *cm*, 4 *cm* and 5 *cm*?

Problem 19. Let p and q be such that $4p + 4q + 1 < 0$. How many real roots does the following equation have: $(x^2 - 2px + q) \times (x^2 - 2qx + p) = 0$?

Задача 20. A motorboat travels a distance of 32 km downstream and 21 km upstream in 3 hours and 21 minutes in total. Find the speed of the boat in calm waters if the speed of the boat when going upstream is equal to 60 % of its speed going downstream.