MATHEMATICS WITHOUT BORDERS

AGE GROUP 8

SPRING 2019

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. For how many odd two-digit numbers $\overline{ab}$ is the number $\sqrt{\overline{ab}}$ a rational number?

Problem 2. If x_1 and x_2 are roots of the equation $x^2 - x - 1 = 0$, calculate

$$|x_1 - x_2|.$$

Problem 3. Calculate $m + n$, if $25x^{m+2n} - x^{2+n}$ is a monomial (has only 1 term).

Problem 4. For which primes $x < 10$ are there only 2 primes that are factors of

$$x^{2019} + 2x^{2018} + x^{2017} ?$$

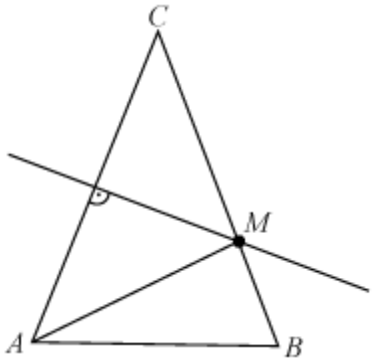
Problem 5. For which natural numbers *n* are $\frac{n+6}{n-1}$, and $\frac{3n+6}{2n-6}$ integers?

Problem 6. The triangle *ABC* is isosceles and *AC* = *BC*. The point L lies on BC such that the line AL bisects $\angle CAB$. If $AC + CL = AB$, find $\angle CBA$.

Problem 7. The equilateral triangles ΔADM and ΔDCN have been built to the outside of the parallelogram $ABCD$ with $\angle BAD = 30^0$ and an area of 10 cm^2. Find the area of ΔMDN in cm^2.

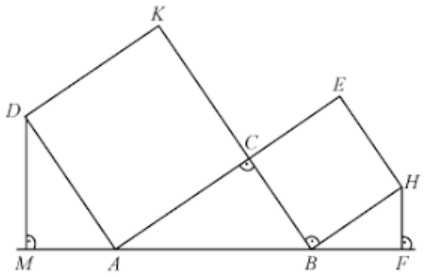
Problem 8. There are 30 dots inside a convex quadrilateral. What is the greatest number of non-overlapping triangles that can be formed that have the given dots and the vertices of the quadrilateral as their vertices?

Problem 9. A line through the side *AC* of the isosceles triangle *ABC* (*AC* = *BC*) intersects the side *BC* at the point *M*. If *CM* = *AM* = *AB*, calculate $\angle ACB$.



Problem 10. Two of the sides of a triangle have lengths of $\sqrt{2}$ *cm* and $\sqrt{3}$ *cm*, respectively. The perpendicular height of one side is 2cm longer than the perpendicular height of the other side. Find the area of the triangle in cm^2.

Problem 11. The diagram below shows ΔACB, which is a right-angled triangle with a hypotenuse AB; and the quadrilaterals *ACKD* and *BCEH*, which are squares. The sides of the right-angled triangle are 3 cm, 4 cm and 5 cm. Find the sum of the distances from the points *D* and *H* to the straight line *AB* extended in cm.



Problem 12. How many zeros are at the end of the smallest number that is divisible both by 2, and by 5, and has 2019 divisors?
Hint: The number 673 is a prime.

Problem 13. Let *A, B* and *C* be integers, such that $(x - A)(x - 2) + 1 = (x + B)(x + C)$ is an identity. Calculate *B* + *C*.

Problem 14. Calculate the sum of the two-digit numbers $\overline{ab}$ and $\overline{ba}$, if

$$600 < (\overline{ab})^2 - \left(\overline{ba}\right)^2 < 700.$$

Problem 15. In how many ways can we separate 9 children into two groups? One of the groups must have 5 children, and the other must have 4.

Problem 16. If $500!! = 2 \times 4 \times \ldots \times 498 \times 500$, find the greatest value of *N*, for which 5^N is a factor of 500!!.

Problem 17. Let *x*, *y* and *z* be natural numbers, such that $|x-y|+|y-1|+|z-2|=3$. Calculate the greatest value of $x+y+z$.

Problem 18. How many numbers *a* are there, where both $(-\frac{3}{a})$ and $(2a-\frac{3}{a})$ are integers?

Problem 19. The digits 3 and 4 were used to form all 4-digit numbers that satisfy the following conditions:

- no two digits 3 are adjacent
- the number must contain both the digit 3 and the digit 4

How many such numbers are there?

Problem 20. The following figure shows an unfolded cube. The faces of the cube have been numbered using the numbers from 1 to 6. Find the greatest sum of the numbers that have been written on three faces that share the same vertex.