MATHEMATICS WITHOUT BORDERS

WINTER 2019

AGE GROUP 7

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. If $|a| = 3$ and $|a - 1| = 1 - a$, calculate $5 - a$.

Problem 2. Find the least value of the natural number n, for which
$2^n + 4^n + 6^n > 1000$ is true.

Problem 3. Calculate the sum of the prime factors of 4433.

Problem 4. Find the midpoint of the line segment with endpoints $-\frac{2}{3}$ and 1.2.

Problem 5. Calculate the sum of all natural numbers n, for which $n - 1$ is a factor
of $n^3 - 4$.

Problem 6. It takes x seconds for a robot to cut off 1 segment (10 cm) from a
metal rod (10 m). There is an interval of x seconds between each two cuts. How
many seconds will it take for x segments of the rod to be cut off, if $x \leq 98$?

Problem 7. There were exactly four Mondays and exactly four Fridays in a
particular January. What day of the week did the 1st of January fall on during that
year?

Problem 8. John wrote down all natural numbers, for which the following is true:
- the number is made up of different even digits;
- the number is smaller than 5000;
- the number is divisible by 18.

How many numbers did John write down?

Problem 9. It is known that the following equation

$$1^2 + 3^2 + 5^2 + \cdots + (2n - 3)^2 + (2n - 1)^2 = \frac{n \times (4n^2 - 1)}{3}$$

is an identity.

Calculate $(1^2 + 3^2 + 5^2 + \cdots + 2017^2 + 2019^2) \div (1010 \times 2019 \times 2021)$.

Problem 10. The sum A of 4 of the following numbers: 36, 1, 2, 3, 4, 5, 7, 8 and 12, is three times greater than the sum B of another set of 4 of these numbers. Calculate $A + B$.

Problem 11. How many line segments are there that connect each 2 vertices of a 20 sided polygon, if these line segments are NOT sides of the 20 sided polygon?

Problem 12. A wooden sphere has been cut into two identical parts. The surface area of one of the resulting solids is 60 cm². Find the surface area of the sphere.

Hint:

The surface area of a sphere with a radius R is $4\pi R^2$.

The area of a circle with a radius r is πr^2.

Problem 13. A rectangle with a perimeter of 60 cm has been divided into two identical rectangles, each of which has a perimeter of 38 cm. Find the area of the original rectangle in centimetres.

Problem 14. 3 blue points, 5 green points and N red points have been marked along a circle. There are 71 segments with endpoints of different colours. Find N.

Problem 15. We have 5 line segments. The first segment intersects 2 of the others. The second one intersects 3 of the others. The third one intersects 4 of the others. The fourth one intersects 4 of the others. Which segments does the fifth segment intersect?

Problem 16. Three metal cubes with edges of 3 cm, 4 cm and 5cm, respectively, have been melted in order to form a new cube. Find the edge of the new cube in centimetres.

Задача 17. Peter added the numbers which belong to the set B $\{-3, -5, 1, 2\}$, but belong neither to A $\{-2, -3, -6, 5\}$, nor to C $\{-1, -2, -3, -6, 2, 5\}$. John added the numbers which belong to A and C, but do not belong to B. By how much is John's sum greater than Peter's sum?

Problem 18. Find the number A, if
$$\frac{1}{1 \times 2} - \frac{5}{2 \times 3} + \frac{7}{3 \times 4} - \frac{9}{4 \times 5} = -\frac{A}{35}.$$

Problem 19. The polynomial $x^5 + x + 1$ can be expressed as the product of the second degree polynomial and the third degree polynomial: $(x^3 - x^2 + 1)$ and $(x^2 + Ax + 1)$.
Find the number A.

Problem 20. A motorboat travels a distance of 32 km downstream and 21 km upstream in 3 hours and 21 minutes in total. Find the speed of the boat in calm waters if the speed of the boat when going upstream is equal to 60 % of its speed going downstream.