MATHEMATICS WITHOUT BORDERS
AGE GROUP 6
SPRING 2019

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Find the tens digit of the number equal to $4 + 4^2 + 4^3 + 4^4$.

Problem 2. Calculate

$$17\frac{4}{17} - 2\frac{4}{35} + 2\frac{13}{17} - 2\frac{31}{35}.$$

Problem 3. How many natural numbers are there that are smaller than 24 and co-primes of 24?

Problem 4. Calculate the value of the following expression:

$$A = -1^5 + (-1)^5 - 2^4 + (-2)^4 - 3^3 + (-3)^3 + (-56) \times (-1).$$

Problem 5. Find the greatest negative integer that has 43 as the sum of its digits.

Problem 6. A prism has 12 faces. How many vertices does it have?

Problem 7. A rectangle has been divided into four smaller rectangles with areas 1, 2, 6 and X. Find the greatest possible value of X.

Problem 8. 4 points have been marked on the square grid below. Three of them have the following coordinates: (-4; 0), (0;0) and (1; 0). Find the x-axis of the fourth point.
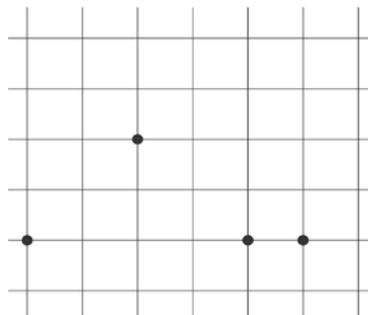
Problem 9. We are given a rectangle with an area of 24 cm^2. The midpoints of its sides are connected in order to form a quadrilateral. Calculate the area of the new quadrilateral in cm^2.

Problem 10. 40 sticks of identical length are necessary to build a 4×4 square grid. How many such sticks would be necessary to build a 10×10 square grid?

Problem 11. Find the remainder left after dividing
$(222 - 2) \times (220 - 4) \times (218 - 6) \times \ldots \times (4 - 220) \times (2 - 222)$ by 111.

Problem 12. Find the number x:
$4 \times 10^5 + 10^4 + x \times 10^2 + 3 \times 10^{-1} + 2 \times 10^{-3} = 411200.302$?

Problem 13. Calculate
$$1 - x + |x - 1| + 2 - x + |x - 2| + \cdots + 21 - x + |x - 21|,$$
if $x = 20.19$.

Problem 14. Ivan wrote down the sums of all even numbers A and B with a product of 40. Find the smallest sum that he wrote down.

Problem 15. Three teams: A, B and C, played a total of 7 games of football against each other. Team A played 3 games and team B played 4 games. How many games did team C play?

Problem 16. A maths test consists of 10 problems. For each correct answer you get 3 points. For each wrong answer you lose 1 point. For each missing answer you get 0 points. How many different scores can you get with this marking system?

Problem 17. The number 1/256000 is presented as a decimal fraction. Find the number of digits after the decimal point.

Problem 18. How many one-digit numbers can be expressed as the sum of two different perfect squares?

Problem 19. The digits 3 and 4 were used to form all 4-digit numbers that fulfil the following criteria:

- two digits 3 cannot be adjacent;
- both the digit 3 and the digit 4 must be part of the number.

How many such numbers are there?

Problem 20. Take a look at the following 9 dots:

$$\begin{matrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{matrix}$$
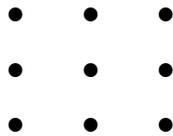
In how many ways can we remove the least number of dots so that no 3 of the remaining dots lie on the same straight line? The order of removing dots is not important.