MATHEMATICS WITHOUT BORDERS

AGE GROUP 9

AUTUMN 2018

INSTRUCTIONS

1. Please DO NOT open the test papers before receiving the proctor's permission.

2. The test contains 20 problems with open answers.

3. You must write down your answers in the ANSWER SHEET.

4. You will get 2 points for each correct answer, 1 point for an incomplete answer, and 0 points for a wrong or missing answer.

5. Using calculators, phones or other electronic devices, as well as books or formula sheets is NOT ALLOWED.

6. You have 60 minutes to complete the test. In the case of two students having the same number of points, the student who completed the test quicker will get a higher ranking place.

7. Taking the test papers and any other notes out of the room is NOT ALLOWED.

8. Receiving any help from a proctor or anyone else during the competition is NOT ALLOWED. The organisers insist on honesty and fair play on the part of all participants in the tournament.

GOOD LUCK!

Arithmetics and algebra

Problem 1. What is the smallest prime which divides the number equal to $3^{20} + 11^{18}$?

Problem 2. Find the prime number *P*, for which (*P* + 64) is a perfect square of a natural number.

Problem 3. Let the positive numbers *x* and *y* be such that

$$\left(2x + \frac{1}{x}\right) \times \left(y + \frac{1}{y}\right) = 4\sqrt{2}.$$

Calculate the integer part of $x - y$.

(Hint: If a and b are nonnegative numbers, then the inequality $a + b \geq 2\sqrt{ab}$ is correct.)

Problem 4. The equality $x^3 + x^2 + a\,x + b = 0$ has a double root 2. Calculate the sum of its three roots.

Problem 5. If $\frac{x}{2x^2+9x+2} = \frac{1}{3}$, calculate $x^5 - 55x - 21$.

Logical Thinking

Problem 6. A few different natural numbers have been written on a white board. Exactly two of them are divisible by 3 and exactly 11 of them are divisible by 11. If *M* is the greatest of the numbers that have been written down, find the smallest possible value of *M*.

Problem 7. Let *a* and *b* are the integer part and the fractional part of $\sqrt{7}$. Calculate the integer part of $a \div b$.

Problem 8. Let Q be a rational number smaller than 10, and I an irrational number greater than 10. If the product of $(Q \times I)$ is a rational number, calculate the possible values of the quotient of $(Q \div I)$?

Problem 9. For how many integers x is the following inequality correct?

$$\frac{x-2}{\sqrt{x+2}} \leq 0?$$

Problem 10. What will be the equation of a straight line which is perpendicular to the *x*-axis and has the point with coordinates (1, 2018) on it?

Geometry

Problem 11. A rectangular parallelepiped has been formed from 4 identical cubes. The surface area of each cube is 2 cm^2. Find the surface area of the rectangular parallelepiped.

Problem 12. We are given a trapezium *ABCD* (*AB* is the small base). The point X is the midpoint of the leg AD и $\measuredangle CXB = 90^0$. If the height of the trapezium is 4 cm, and the leg *BC* has a length of 6 cm, calculate the area of the trapezium in square centimeters.

Problem 13. We are given a rectangle with side lengths of 6 cm and 11 cm. The bisectors of the angles that belong to one of the longer sides divide the opposite side into three parts. Find the length of the smallest of these three parts in centimeters.

Problem 14. The sides of a cube with an edge of 8 cm were painted. After that the cube was divided into smaller cubes with an edge of 1 cm. How many of the smaller cubes have at least one painted side?

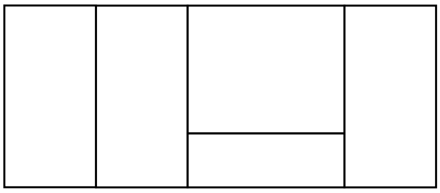
Problem 15. We are given a right-angled triangle with side lengths of 6 cm, 8 cm and 10 cm. By how many centimeters is the radius of the circumscribed circle greater than the radius of the inscribed circle?

Combinatorics

Problem 16. Find the sum of the ten-digit numbers that have 2 as the sum of their digits.

Problem 17. The sum of six odd natural numbers is 20. Find the smallest possible difference of the greatest and the smallest among these numbers.

Problem 18. The figure on the diagram is made up of 5 rectangles. We must paint them in 5 colors. Each two neighboring rectangles must NOT be painted in the same color. In how many ways can we paint the rectangles if it is not necessary to use all available colors?



Problem 19. Three people had a few pieces of fruit of different weights. They distributed the fruit between themselves so that each of them received the same weight of fruit (without having to cut the fruit). After that, another two people joined them and they had to redistribute the fruit. Each of them received the same amount of fruit again, without having to cut the fruit. At least how many pieces of fruit did they have in the first place?

Problem 20. Let us observe the following numbers:

$$a_1, a_2\ , a_1 \times a_2, a_3, a_4\ , a_3 \times a_4, \ldots., a_{2017}, a_{2018}\ , a_{2017} \times a_{2018}.$$

What is the greatest possible number of negative numbers among them?