MATHEMATICS WITHOUT BORDERS

AGE GROUP 8

AUTUMN 2018

INSTRUCTIONS

1. Please DO NOT open the test papers before receiving the proctor's permission.

2. The test contains 20 problems with open answers.

3. You must write down your answers in the ANSWER SHEET.

4. You will get 2 points for each correct answer, 1 point for an incomplete answer, and 0 points for a wrong or missing answer.

5. Using calculators, phones or other electronic devices, as well as books or formula sheets is NOT ALLOWED.

6. You have 60 minutes to complete the test. In the case of two students having the same number of points, the student who completed the test quicker will get a higher ranking place.

7. Taking the test papers and any other notes out of the room is NOT ALLOWED.

8. Receiving any help from a proctor or anyone else during the competition is NOT ALLOWED. The organisers insist on honesty and fair play on the part of all participants in the tournament.

GOOD LUCK!

Arithmetics and algebra

Problem 1. What is the smallest prime that divides the number equal to

$$3^{20} + 5^{18}?$$

Problem 2. Let a and b be integers, for which $|a| < 3$ and $1 < |b| < 5$.
For which value of b does the expression $a - 3b$ attain the greatest value?

Problem 3. Let the natural numbers x and y be such that

$$\left(x + \frac{1}{x}\right) \times \left(2y + \frac{1}{y}\right) = 6.$$

Calculate $2x + y$.

Problem 4. An object traveled a distance of 60 km at a speed of 30 km/h. Then it traveled a further 40 km at a speed of X km/h. Calculate X, if the object's average speed is 40 km/h.

Problem 5. If $\frac{x}{2x^2+9x+2} = \frac{1}{3}$, calculate $x^4 + 21x + 100$.

Logical Thinking

Problem 6. Someone asked Pythagoras about the time, and his reply was as follows:
"The time left until the end of the day is equal to two times two fifths of the time which has already passed." (twenty-four-hour period)
What's the time?

Problem 7. In how many ways can we pour 38 liters of juice in a total 10 bottles, each one with a capacity either of 1 liter, 3 liters or 5 liters by using all three types of bottles?

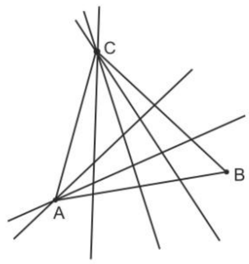
Problem 8. The product of six primes is 240. Find the sum of these numbers.

Problem 9. The average age of x boys is 13 years, and the average age of $2 \times x$ girls is 10. What is the average age of these girls and boys? *(x is a natural number)*

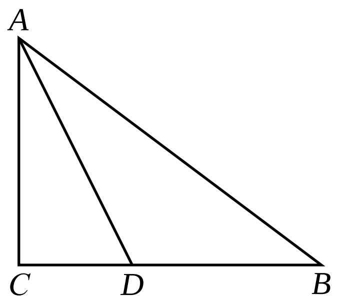
Problem 10. There are more than 5 members in an athletics club. The girls are more than 93%. At least how many girls are there in this club?

Geometry

Problem 11. We are given the triangle *ABC*. A number of straight lines have been drawn through two of its vertices. The straight lines cross the opposite sides. In this way the triangle has been divided into 12 parts that do not intersect. If we build *X* straight lines through one of the vertices and 2*X* + 1 straight lines through another vertex, we will get 800 parts.
Calculate *X*.



Problem 12. The triangle *ABC* is a right-angled triangle with a right angle at the vertex *C* and side lengths of 3 cm, 4 cm and 5 cm. The angle bisector of angle *A* intersects the side *BC* at point *D*. At least how many square centimeters is the area of the triangle *ABD?*



Problem 13. The faces of a cube with an edge length of 8 cm were painted. After that the cube was divided into smaller cubes with an edge length of 1 cm. How many of the smaller cubes have at least one painted face?

Problem 14. The segment *AB* has a length of 72 cm. Annie placed dividing points along the segment in order to create 8 equal parts. Peter also placed dividing points along the segment in order to create 12 equal parts. How many dividing points have been placed along the segment *AB*?

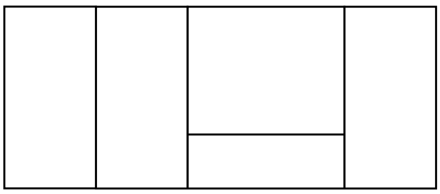
Problem 15. The median and the height to the hypotenuse of a right-angled triangle are 4 *cm* and 2 *cm*, respectively. How many degrees is the greatest acute angle of this triangle?

Combinatorics

Problem 16. How many ten-digit numbers are there that have 2 as the sum of their digits?

Problem 17. The sum of six odd natural numbers is 20. Find the smallest possible difference of the greatest and the smallest of these numbers.

Problem 18. The figure below is made up of 5 rectangles. We must paint them in 5 colors. No two neighboring rectangles can be painted in the same color. In how many ways can we paint the rectangles if it is not necessary to use all available colors?



Problem 19. Three people had a few pieces of fruit of different weights. They distributed the fruit between themselves so that each of them received the same weight of fruit (without having to cut the fruit). After that, another two people joined them and they had to redistribute the fruit. Each of them received the same amount of fruit again, without having to cut the fruit. At least how many pieces of fruit did they have in the first place?

Problem 20. Take a look at the following numbers:

$a_1, a_2 , a_1 \times a_2, a_3, a_4 , a_3 \times a_4, \ldots, a_{2017}, a_{2018} , a_{2017} \times a_{2018}$.

What is the greatest possible number of negative numbers among them?