



MATHEMATICS WITHOUT BORDERS

AGE GROUP 7

AUTUMN 2018

INSTRUCTIONS

1. Please DO NOT open the test papers before receiving the proctor's permission.
2. The test contains 20 problems with open answers.
3. You must write down your answers in the ANSWER SHEET.
4. You will get 2 points for each correct answer, 1 point for an incomplete answer, and 0 points for a wrong or missing answer.
5. Using calculators, phones or other electronic devices, as well as books or formula sheets is NOT ALLOWED.
6. You have 60 minutes to complete the test. In the case of two students having the same number of points, the student who completed the test quicker will get a higher ranking place.
7. Taking the test papers and any other notes out of the room is NOT ALLOWED.
8. Receiving any help from a proctor or anyone else during the competition is NOT ALLOWED. The organisers insist on honesty and fair play on the part of all participants in the tournament.

GOOD LUCK!

Arithmetics

Problem 1. Let us subtract the quotient of the numbers (-18) and 6 from the product of the numbers X and (-6). If the difference is 15, find the number X .

Problem 2. The product of four integers is 7, and their sum is 6. Which is the smallest among these four numbers?

Product 3. Calculate

$$\left(\frac{1}{2} - 1\right) \div \left(\frac{1}{3} - 1\right) \div \left(\frac{1}{4} - 1\right) \div \left(\frac{1}{5} - 1\right) \times \left(\frac{1}{6} - 1\right) + \frac{1}{24}$$

Product 4. Which is the greatest integer that is not greater than the following number?

$$\frac{1}{3} - \frac{8}{15} + \frac{12}{35} - \frac{16}{63}$$

Problem 5. For how many primes N does the number equal to 2^N divide the product of all natural numbers from 1 to 30 with a remainder of 0?

Logical Thinking

Problem 6. We have 6 identical pairs of black shoes, 7 identical pairs of brown shoes and 8 identical pairs of grey shoes. I'm trying to choose 2 pairs of shoes of a different color, without looking. At least how many shoes should I take in order to do that?

Problem 7. Two sons, two fathers and one grandson are traveling in a car. At least how many people are traveling in this car?

Problem 8. Three children – Alec, Boris and Kate won a gold, a silver, and a bronze medal from a competition in mathematics. The three children are from three different schools: № 1, № 2 and № 3.

- Alec is not from school № 1;
- Boris is not from school № 3;
- The student from school № 1 has not won the silver medal;
- Boris is not a gold medalist;
- The student from school № 3 won a bronze medal.

Which school is Kate from?

Problem 9. At least how many symbols „+“ should we place to the left of the following expression, in order to get a correct numerical equality?

$$\underbrace{3333\dots3}_{25 \text{ digits } 3} = 2019$$

Problem 10. In six boxes there are 15, 16, 18, 19, 20 and 31 coins, respectively. Ivan and Peter took a total of 5 boxes. The number of coins that one of them took is twice as much as the number of coins the other took. Steven took the sixth box. How many coins are there in Steven's box?

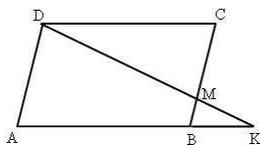
Geometry

Problem 11. Calculate the area of the triangle ABC , if $A(1,0)$, $B(2,1)$ and $C(0,3)$.

Problem 12. A rectangular parallelepiped (box) has a volume of 99 cm^3 . If the lengths of its edges are expressed in integers greater than 1 (in centimeters), how many square centimeters is the total surface area of this parallelepiped?

Problem 13. The faces of a cube with an edge of 6 cm were painted and after that the cube was divided into smaller cubes with an edge of 1 cm. How many of the smaller cubes have at least one painted face?

Problem 14. The area of the parallelogram $ABCD$ is 18 cm^2 . The point M divides the side BC in a 1:2 ratio, counting from the point B . Calculate the area of $\triangle BMK$.



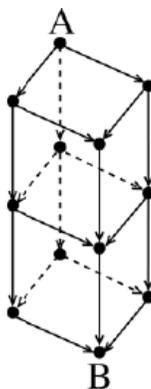
Problem 15. Two of the sides of a triangle have lengths of 5 cm and 6 cm, respectively. One of the heights towards them is 2 cm longer than the other. Calculate the area of the triangle in square centimeters.

Combinatorics

Problem 16. Four children met together: Adam, Bobby, Charlie and Daniel. Adam shook hands with 3 of the children, and Bobby shook hands with 2. Daniel shook hands with less children than Charlie. How many children did Daniel shake hands with?

Problem 17. How many digits are there in the smallest natural number that only contains the digits 0 and 1 and is divisible by 180?

Problem 18. In how many ways can we get from A to B if we can only move along the arrows?



Problem 19. How many triangles are there whose vertex coordinates are nonnegative integers smaller than 3?

Problem 20. How many three digit numbers have 24 as the sum of their digits?