MATHEMATICS WITHOUT BORDERS

AGE GROUP 6

AUTUMN 2018

INSTRUCTIONS

1. Please DO NOT open the test papers before receiving the proctor's permission.

2. The test contains 20 problems with open answers.

3. You must write down your answers in the ANSWER SHEET.

4. You will get 2 points for each correct answer, 1 point for an incomplete answer, and 0 points for a wrong or missing answer.

5. Using calculators, phones or other electronic devices, as well as books or formula sheets is NOT ALLOWED.

6. You have 60 minutes to complete the test. In the case of two students having the same number of points, the student who completed the test quicker will get a higher ranking place.

7. Taking the test papers and any other notes out of the room is NOT ALLOWED.

8. Receiving any help from a proctor or anyone else during the competition is NOT ALLOWED. The organisers insist on honesty and fair play on the part of all participants in the tournament.

GOOD LUCK!

Arithmetics

Problem 1. Calculate

$$\frac{1+2\times 2+3\times 3+4\times 4}{2\times 2+4\times 4+6\times 6+8\times 8}$$

Problem 2. How many integers are there from 0 to 10 that have an odd number of positive divisors?

Problem 3. Calculate the sum of $0.0025 \div 50$ and 1.99995.

Problem 4. The least common multiple of two natural numbers is 60, and the greatest common divisor of the same numbers is 6. Calculate the sum of these numbers.

Problem 5. Ivan started writing the odd numbers from 1 onwards in a row on a whiteboard. He stopped when two digits 9 appeared next to each other on the board. How many digits did he write before the two digits 9?

Logical Thinking

Problem 6. What is the smallest three-digit number that, when divided by 4, 5 and 6, leaves a remainder of 3?

Problem 7. A salesman bought some items from the wholesale market and determined a price at which to sell the items in his own store in order to make 20% profit. Later on he lowered the price by 10% and sold the items at the new price. What % profit did he get in the end?

Problem 8. Ivan wrote down all natural numbers from 1 to 201 (including those two numbers). Then he removed all numbers that are divisible by 3 and by 5. How many numbers are left?

Problem 9. At least how many symbols „+“ should we place to the left of the following expression in order for the numerical equality to be correct?

$$\underbrace{222\ldots2}_{37\text{ digits }2} = 2018$$

Problem 10. Find the number of irreducible proper fractions with 18 as their denominator.

Geometry

Problem 11. What is the greatest possible number of rectangles that can be formed using 10 straight lines?

Problem 12. The diagonals *AC* and *BD* of the quadrilateral *ABCD* are mutually perpendicular and have lengths of 10 cm and 8 cm, respectively. If the diagonal *AC* bisects the diagonal *BD*, calculate the area of the quadrilateral *ABCD* in square centimeters.

Problem 13. The faces of a cube with an edge of 6 cm were painted and then the cube was divided into smaller cubes with edges of 1 cm. How many of the smaller cubes have at least one painted face?

Problem 14. The points *A, B* and *C* lie on a straight line in such a way that:

- The distance from point *A* to point *B* is 6 cm*;*
- The distance from point *C* to point *A* is 2 cm*;*
- The distance between the midpoints of the segments *AB* and *AC* is 2 cm.

Find the length of the segment BC in centimeters.

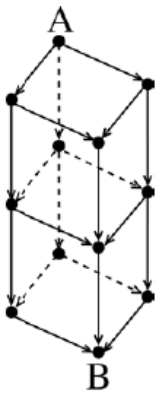
Problem 15. There are three isosceles triangles with lengths expressed in integer centimeters and a perimeter of 16 cm. Find the smallest side of these triangles in centimeters.
(Hint: The sum of any two side lengths of a triangle is always greater than the third side.)

Combinatorics

Problem 16. We are given three-digit numbers. If we only remove the first digit of each of those numbers, we would get a perfect square. If we only remove the last digit of each of those numbers, we would get a prime. How many such three-digit numbers are there?

Problem 17. How many digits are required to write down the smallest natural number that only contains the digits 0 and 1 and is divisible by 72?

Problem 18. In how many ways can we go from point A to point B if we are only allowed to move along the arrows?



Problem 19. The points *A, B, C* and *D* lie on a straight line. Another straight line passes through point D, along which lie another 3 points: *X, Y* and *Z*. How many triangles have their vertices among the points?

Problem 20. Which number should we place in □, so that the following equality would be correct?

$$\frac{11+\square}{41+\square}=\frac{3}{8}$$