MATHEMATICS WITHOUT BORDERS

2015-2016

AUTUMN 2015: GROUP 5

Problem 1. Which one is the smallest product among the following?

A) $5 \times 13 \times 35$ **B)** $5 \times 14 \times 34$ **C)** $5 \times 12 \times 36$ **D)** $5 \times 14 \times 36$

Problem 2. The number of integers from 98 to 1,000 which are divisible by 3 is:

A) 301 **B)** 302 **C)** 303 **D)** 304

Problem 3. If we have a 200 cm long tape and we cut off 12 dm from it, how long is the larger part of the two cut-offs?

A) 188 cm **B)** 80 cm **C)** 8 dm **D)** 12 dm

Problem 4. Peter reads 15 pages in 45 minutes. How long will it take him to read 45 pages at that rate?

A) 15 mim **B)** 1 h **C)** 2 h 15 min **D)** 1 h 15 min

Problem 5. The sum of the first 100 positive integers is 5,050. Find the sum of the first 100 positive odd integers.

A) 10,000 **B)** 10,050 **C)** 10,100 **D)** 10,150

Problem 6. There were a total of 90 coins in two boxes. Ten coins were then shifted from the first box to the second. As a result, the number of coins in the second box was twice as much as the number of coins in the first one. What was the number of coins in the first box before the shift?

A) 100 **B)** 80 **C)** 60 **D)** 40

Problem 7. In $A + B + \overline{ABC} = \overline{DEEF}$ each letter corresponds to a digit. Identical letters correspond to identical digits and different letters correspond to different numbers. What is the greatest possible number that corresponds to $\overline{DEEF}$?

A) 1007 **B)** 1006 **C)** 1005 **D)** 1004

Problem 8. What is the ones digit of the smallest natural number with sum of its digits equal to 2015?

A) 9 **B)** 8 **C)** 7 **D)** 6

Problem 9. The number of minutes in 6 hours is the same as the number of hours in:

A) 8 days **B)** 10 days **C)** 15 days **D)** 20 days

Problem 10. How many squares are there in the figure below?

A) 12 **B)** 20 **C)** 22 **D)** 24

Problem 11. Find the value of

$$3 + 6 + 9 - 12 + 15 + 18 + 21 - 24 + \cdots + 51 + 54 + 57 - 60 + 63 + 66 + 69 - 72.$$

Problem 12. We have written down the numbers that are divisible by 5: 5, 10, 15, 20, 25, ... Underneath each of these numbers (in a second row) we have written down the sum of its digits. Which place in the second row will be occupied by the number 14 for the first time?

Problem 13. When multiplying two numbers, Amy miswrote one of the factors: instead of 24 she wrote 42, and got a product of 714. What should be the correct product?

Problem 14. In a group of 60 people, 35 have brown hair, 30 have brown eyes and 20 have both brown hair and brown eys. How many have neither brown hair nor brown eyes?

Problem 15. How many of the products of the numerical sequence

$1 \times 2 \times 3\,, 2 \times 3 \times 4\,, 3 \times 4 \times 5\,, 4 \times 5 \times 6\,, \ldots, 98 \times 99 \times 100$ are divisible by 6?

Problem 16. A square is divided into 9 squares. The square R is coloured in red. Each of the remaining squares is coloured either in red (R), blue (B) or green (G). If in each row and in each column the squares are coloured in all three colors, what would the colour of the X square be?

R		
		X
G		

Problem 17. In a certain year, January had exactly four Tuesdays and four Saturdays. On what day did January 1 fall that year?

Problem 18. 111 111 111 divided by 9 equals $\overline{1234567x}$. Find the x.

Problem 19. If $A = 3 : 3 + 6 : 3 + 9 : 3 + \cdots + 24 : 3 + 27 : 3$, find $A : 45$.

Problem 20. In two rooms there were a total of 45 people. After 15 people left the first room and 20 people left the second room, the number of people remaining in both rooms was equal. How many people remained in the first room?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	C) 5×12×36	We compare 5×455; 5×476; 5×432; 5×504
2	A) 301	The first number is $99 = 3 \times 33$, and the last is $999 = 3 \times 333$. The number is $333 - 32 = 301$.
3	D) 12	200 – 120 = 80 см; 80 см < 12 dm
4	C)	45 can be read in 135 minutes, which is equivalent to 2 hours and 15 minutes
5	A) 10,000	$1 + 2 + 3 + \cdots + 99 + 100 =$ $= \underbrace{(1 + 100) + (2 + 99) + \cdots + (50 + 51)}_{50} = 50 \times 101 = 5050$ 1+3+5+... +197+199=$\underbrace{(1 + 199) + (3 + 197) + \cdots + (99 + 101)}_{50} =$ $50 \times 200 = 10000$ *Another way*: 1+(2 +99)+3+(4+99)+...+99+(100+99) =5050 + 50×99=5050+4950=10 000.
6	D) 40	After moving the coins, in the first box there are 30 coins, and in the second box there are 60 coins. Before that there were 40 coins in the first box and 50 in the second box.
7	D) 1004	If A<9, then $A + B + \overline{ABC} \leq 8 + 9 + \overline{89C} < 910 < 1000$ In this case A=9. If B<8, then $9 + B + \overline{9BC} \leq 9 + 7 + \overline{97C}$<1000 In this case B=9. $A + B + \overline{ABC} = 9 + 8 + \overline{98C} = 997 + C$. The possible values of C are the digits 0, 1, 2,3, 4, 5, 6 and 7. 997+C would be a four digit number if C=3, 4, 5, 6 and 7. Only for C=5, 6 and 7, we get the four digit number $\overline{DEEF}$. The greatest of them is 1004, which we get from C=7.
8	A) 9	The smallest number consists of the smallest amount of digits, therefore the predominant digits are 9. From 2015÷9= 223 (remainder 8), it follows that the smallest number is $\underbrace{8999 \ldots 9}_{224\ digits}$.

9	C) 15	The minutes in 6 hours are $6 \times 60 = 360$. The hours in a day are 24. 360 ÷ 24 = 15
10	B) 20	12 squares with a side of 1; 6 squares with a side of 2; 2 squares with a side of 3;
11	396	$\underbrace{(3+6+9-12)+(15+18+21-24)+\cdots+(63+66+69-72)}_{6\ groups}$ $== 6+30+54+78+102+126 = 396.$ $3+6+9-12+15+18+21-24+\cdots+51+54+57-60+$ $63+66+69-72=$ $=3+6+9+12+15+18+21+24+\cdots+51+54+57+60+$ $63+66+69+72-2\times(12+24+36+48+60+72)=$ $=12\times 75-2\times 3\times 84 = 900-504 = 396.$
12	19	$5, \underbrace{1, 6, 2, 7, 3, 8, 4, 9, \ldots 8, 13, 9, 14}_{18}$
13	408	714 ÷ 42 = 17; 17 × 24 = 408
14	15	With brown hair, but not with brown eyes: 35 – 20 = 15; With brown eyes, but not with brown hair: 30 – 20 = 10. With brown eyes and with brown hair: 20. We need to deduct 15 + 10 + 20 = 45 from the total number, in order to find the number of people that have neither brown hair nor brown eyes: 60 – 45 = 15.
15	98	All products are divisible by 6. It is known that the product of two consecutive numbers is divisible by 1×2; of three consecutive numbers – by $1 \times 2 \times 3$, of four consecutive numbers – by $1 \times 2 \times 3 \times 4$, etc.
16	Red or green	R B G R G B B G R B R G G R B G B R
17	Wednesday	January has 4 full weeks and 3 days (31 days). Let us assume that a Sunday and a Monday have been included in the fifth week. In this case the fifth week would include either a Saturday or a Tuesday. The Saturdays are now 5. Therefore the three days are

		Wednesday, Thursday and Friday. January 1 is on Wednesday. *Other way* If January 1 is: <u>On Monday:</u> The Mondays would be – 5; Tuesdays – 5, Wednesdays – 5; Thursdays – 4, Fridays – 4; Saturdays – 4, Sundays – 4; <u>On Tuesday:</u> The Mondays would be – 4; Tuesdays – 5, Wednesdays – 5; Thursdays – 5, Fridays – 4; Saturdays – 4, Sundays – 4; <u>On Wednesday:</u> The Mondays would be – 4; Tuesdays– 4, Wednesdays – 5; Thursdays– 5, Fridays– 5; Saturdays – 4, Sundays – 4; <u>On Thursday:</u> The Mondays would be – 4; Tuesdays – 4, Wednesdays – 4; Thursdays – 5, Fridays– 5; Saturdays – 5, Sundays – 4; <u>On Friday:</u> The Mondays would be – 4; Tuesdays – 4, Wednesdays – 4; Thursdays – 4, Fridays – 5; Saturdays – 5, Sundays – 5; <u>On Saturday:</u> The Mondays would be – 5; Tuesdays – 4, Wednesdays – 4; Thursdays – 4; Fridays – 4; Saturdays– 5, Sundays– 5; <u>On Sunday:</u> The Mondays would be – 5; Tuesdays – 5, Wednesdays – 4; Thursdays – 4; Fridays – 4; Saturdays – 4, Sundays – 5;.
18	9	12345679×9=111 111 111
19	1	$A = 3 : 3 + 6 : 3 + 9 : 3 + \cdots 24 : 3 + 27 : 3 =$ $= 1 + 2 + \cdots 8 + 9 = 45$
20	5	45 – (15 + 20) = 10; 10÷2 = 5.

WINTER 2016: GROUP 5

Problem 1. $223 \times 4 - 110 \times 4 \ - 112 \times 4 =$

A) 0 **B)** 4 **C)** 8 **D)** other

Problem 2. Which of the following products is the greatest?

A) $123 \times 5 \times 7$ **B)** $123 \times 5 \times 6$ **C)** $123 \times 6 \times 7$ **D)** $123 \times 5 \times 8$

Problem 3. The sum of five different odd natural numbers is 27. Which of these numbers is the greatest?

A) 5 **B)** 7 **C)** 9 **D)** 11

Problem 4. How many even numbers are there from 205 to 2,017?

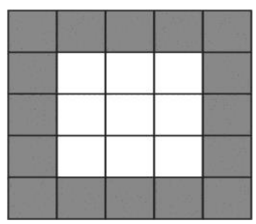
A) 1,812 **B)** 1,813 **C)** 907 **D)** 906

Problem 5. If we diminish the dividend 10 times and increase the divisor 10 times, what would happen to the quotient?

A) it will be diminished 100 times **B)** it will not change

C) it will be diminished 10 times **D)** it will increase 10 times

Problem 6. By how many are the non-coloured squares less than the coloured squares?



A) 8 **B)** 6 **C)** 4 **D)** 2

Problem 7. Find the difference of the smallest number that is greater than 2,016 and has the same sum of its digits as 2,016, and the number 2,016.

A) 9 **B)** 7 **C)** 5 **D)** 3

Problem 8. Steve had a bowl with some sweets in it. At first he ate a third of the sweets. After that he ate a fourth of what was left in the bowl. In the end, he ate a sixth of the remaining sweets. At this point there were 10 sweets left in the bowl. How many sweets were there in the beginning?

A) 27 **B)** 24 **C)** 21 **D)** 18

Problem 9. What is the sum of the missing digits in the following equation?

$$7* + **6 + *93 = 2{,}016$$

A) 29 **B)** 27 **C)** 24 **D)** 18

Problem 10. What is the number of all six-digit numbers $\overline{20a16b}$, , that are divisible by 5 (with a remainder of 0), and are recorded using different digits? (*a* and *b* are digits.)

A) 2 **B)** 5 **C)** 10 **D)** 15

Problem 11. The natural number *A* would be increased 11 times if, on its right side, we write down one of the following nine digits: 1, 2, 3, 4, 5, 6, 7, 8 or 9. How many digits does the number *A* have?

Problem 12. In how many different ways can we divide a set of 7 different weights (from 1 to 7 grams each) in two groups of equal weight?

Problem 13. Place the digits 0, 1, 2 and 3 in the squares in such a way that would result in the greatest possible product. What is the product?

□□×□□

Problem 14. When x is divided by 55 the remainder is 22. What is the remainder when $3 \times x$ is divided by 55?

Problem 15. A rectangle has a width of 18 *cm*, and a length four times greater than the width. How many *dm* is the parameter of the rectangle?

Problem 16. $999 \div 4 + 111 \div 4 - 102 \div 4 = ?$

Problem 17. One of the three brothers *A*, *B* and *C* took the golden apple. Their father asked them who took it and they answered as follows:

A: "*B* took the golden apple."

B: "I took the golden apple."

C: "*A* took the golden apple."

Who actually took the golden apple, if only one of the three brothers was telling the truth?

Problem 18. How many odd natural numbers that are smaller than 15 can be presented as a sum of two prime numbers?

(*Hint:* A *prime* number is a number, larger than 1, that can only be divided evenly by itself and 1. For example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...)

Problem 19. There were 18 apples in one basket, and 20 apples in another basket. I took a few apples from the first basket, and then I took as many apples as were left in the first basket, from the second basket. How many apples in total are left in both baskets?

Problem 20. With a single jump, a grasshopper  can move by 8 *cm* and 1 *mm*, and with a single step, a turtle  can move by 8 *mm*. How many steps will it take for the turtle to go the same distance that the grasshopper would have gone after 8 jumps?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**B**	*First way:* $892-440-448=892-888=4$. *Second way:* $(223-110-112)\times 4 = 1\times 4 = 4$.
2	**C**	$123\times 5\times 7 > 123\times 5\times 6$ $123\times 5\times 7 < 123\times 6\times 7$ $123\times 6\times 7 = 123\times 42 > 123\times 40 = 123\times 5\times 8$ The greatest product is $123\times 6\times 7$
3	**D**	$1 + 3 + 5 + 7 + 9 = 25$, therefore the numbers are 1, 3, 5, 7 and 11. The greatest of these numbers is 11.
4	**D**	*First way:* Let us observe the pairs of numbers – odd, even (205, 206); (207, 208);…; (2015, 2016); (2017, 2018). The pairs are 907. In each of them there is an even and an odd number. The odd numbers are 907, and the even numbers from 205 to 2017 are 906. *Second way:* The even numbers from 205 to 2017 can be presented as follows: $206 + \mathbf{0}\times 2$; $206 + \mathbf{1}\times 2$; $206 + 2\times 2$; ...; $206 + \mathbf{905}\times 2 = 2016$
5	**A**	$(a \div 10) \div (b\times 10) = (a \div b) \div 100$
6	**D**	The coloured squares are 16. The non-coloured squares are 14: 9 squares with a size of 1×1, 4 squares with a size of 2×2, 1 square with a size of 3×3.
7	**A**	The smallest number we are looking for is 2,025. The difference we are looking for is 9.
8	**B**	We can reach the solution by carrying out a check by using the possible answers. If the sweets are 24, then Steve would have eaten 8 sweets, and there would have been 16 sweets left. After that he would have eaten 4, and 12 sweets would have been left. In the end he would have eaten 2, and 10 sweets would have been left.
9	**A**	7**7** + **94**6 + **9**93 = 2016 The sum of the missing digits is 29.

10	**B**	The digit *b* is equal to 5. The digit *a* is equal to 3, 4, 7, 8 or 9.
11	**1**	If we present the number as $\overline{Ax}$, *x* being one of the digits from 1 to 9, then $10A + x = 11A$. Therefore $x=A$, i.e. *A* is a one-digit number.
12	**4**	The total weight of the set of weights is 28 grams. If the weight of 7 grams is in the first group, then we would need to add other weights in the group that have a total weight of 7 grams. 7 = 6 + 1 = 2 + 5 = 3 + 4 = 1 + 2 + 4, therefore we could group them in 4 ways. **First way:** first group – 7, 6, 1; second group – 2, 3, 4, 5 **Second way:** first group – 7, 5, 2; second group – 1, 3, 4, 6 **Third way:** first group – 7, 4, 3; second group – 1, 2, 5, 6 **Fourth way:** first group – 7, 4, 2, 1; second group – 3, 5, 6.
13	**630**	$30 \times 21 = 630$
14	**11**	The number is $55A + 22$. Therefore $3 \times x$ is $165A + 66 = 55\,(3A+1) + 11$.
15	**18**	The length is 72 *cm*. The parameter is 180 *cm* = 18 *dm*.
16	**252**	$(999 + 111 - 102) \div 4 = 252$.
17	**A**	A and B claim the same thing. From the condition of the problem it follows that they are not telling the truth. Only C is telling the truth.
18	**4**	5 = 2 + 3; 7 = 5 + 2; 9 = 7 + 2; 13 = 11 + 2
19	**20**	<u>*First way:*</u> If we take 2 apples, then the apples left in the first basket would be 16. Then if we take 16 apples from the second basket, 4 apples will be left there. The total number of apples left is 20. <u>*Second way:*</u> If we take *x* number of apples from the first basket, then the apples left in it would be $18 - x$. Then if we take $18 - x$ from the second basket, there would be $20 - (18 - x) = 2 + x$ apples left in it. The total number of apples left is $18 - x + 2 + x = 20$.
20	**81**	The grasshopper will go a distance of 8×81 *mm*, and the turtle would go a distance of $x \times 8$ *mm*. $x \times 8 = 8 \times 81$ $x = 81$

SPRING 2016: GROUP 5

Problem 1. The tens and hundredths digits in the number *A* were reversed and the resulting number turned out to be 20.16. What was the original number?

A) 60.12 **B)** 61.02 **C)** 20.61 **D)** 10.26

Problem 2. Instead of being reduced 10 times, a number was increased 10 times and the result was 20.16. What is the number that was meant to be received?

A) 201.6 **B)** 2.016 **C)** 0.2016 **D)** other

Problem 3. *A* is the smallest natural number, so that when divided by 9 leaves a remainder of 6. What would be the remainder if the number *A* is divided by 4?

A) 0 **B)** 1 **C)** 2 **D)** 3

Problem 4. The speed of a boat going along the stream is 18 *km/h*, and the speed of the same boat going against the stream is 12 *km/h*. What is the speed of the boat in still water?

A) 13 *km/h* **B)** 14 *km/h* **C)** 15 *km/h* **D)** 30 *km/h*

Problem 5. Calculate the value of the expression

$$\frac{4-2}{2\times 4}+\frac{6-4}{4\times 6}+\frac{8-6}{6\times 8}.$$

A) 0.375 **B)** 0.275 **C)** 0.125 **D)** 0.1

Problem 6. The number of positive integers from 1 to 2,016, which cannot be divided by 2 or 5, is:

A) 1210 **B)** 1008 **C)** 202 **D)** 806

Problem 7. How many proper irreducible fractions are there, which have a one-digit denominator and a numerator other than 0?

A) 25 **B)** 27 **C)** 30 **D)** 35

Problem 8. If the square is magical, find X.

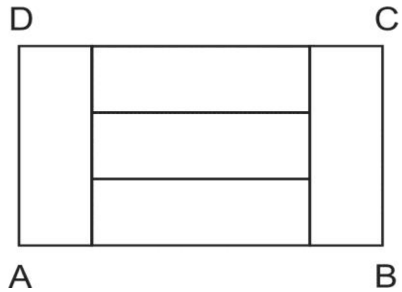
$\frac{2}{3}$		
	$1\frac{2}{3}$	X
$1\frac{1}{3}$		

A) $\frac{1}{3}$ **B)** $\frac{3}{12}$ **C)** $\frac{1}{5}$ **D)** $\frac{12}{15}$

Problem 9. The number $\overline{12a34a56a78a}$ consists of 12 digits (1, 2, 3, ..., 8 and the number a is included 4 times), and it is divisible by 3 and 5. What is the digit a?

A) 0 **B)** 3 **C)** 5 **D)** 9

Problem 10. The rectangle *ABCD* below is made up of five identical rectangles. How many square centimeters is the area of the rectangle *ABCD* equal to, if *BC* = 1.5 *cm*?



A) 3.75 **B)** 4.75 **C)** 3.5 **D)** 3

Problem 11. If 1 + 12 + 123 + 1,234 +...+ 12,345,678 + 123,456,789 =$\overline{\ldots abc}$, then $\overline{abc}$ =...

Problem 12. We have 5 identical chocolate bars, each consisting of 28 pieces. We have to divide them equally between 7 children. What is the minimum number of times we need to break each chocolate bar in order to do this?

Problem 13. The natural number *A* has 3 divisors (natural numbers, including 1 and the number *A* itself), the natural number *B* has 2 divisors (natural numbers, including 1 and the number *B* itself), and the smallest common denominator of the two numbers is 9. How many natural numbers are there which are divisors of the number equal to *A* + *B* (including 1 and the number itself)?

Problem 14. All four-digit numbers containing the 4 digits 0, 1, 2 and 6 have been written down. By how many are the numbers greater than 2016 more than the numbers smaller than 2016?

Problem 15. A book has been numbered as follows: the pages of the first sheet have been numbered as 1 and 2; the pages of the second - as 3 and 4, and so on, until the pages of the last sheet, which have been numbered as 227 and 228. If I open the book at a random place, how many possible two-digit numbers are there, which are a product of the digits of the pages that I am currently on?

Problem 16. Four children met together: Adam, Bobby, Charley and Daniel. Adam shook hands with 3 of these children, Bobby shook hands with 2, and Charley shook hands with 1. How many of the children's hands did David shake?

Problem 17. In a sports club there are 12 gold, 14 silver and 13 bronze medalists. There are 30 individual medalists in total in the club and each of them has at least one medal. None of the gold medalists has a silver medal, but 5 of them have bronze medals too. How many of the bronze medalists also have silver medals?

Problem 18. Annie has a magical necklace. Each bead of the necklace is numbered (1, 2, 3, 4 and so on). If between the beads numbered as 5 and 15 there is the same number of beads, what is the total number of beads on Annie's necklace?

Problem 19. The expression we are going to use for this problem is

$$6 \div 2 + 4 \times 3 - 1 \times 10.$$

Replace one of the numbers from the expression with such a number that the initial value of the expression would be increased by 1. How many of the numbers can be changed?

Problem 20. There are 9 coins, one of which is fake and it is lighter. What is the least possible number of times we would have to weigh the coins (using scales) in order to find the fake one?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**A**	In the number 20.16 the digit of tens is 2, and the digit of hundredths is 6. Therefore the number we are looking for is 60.12.
2	**C**	The number 20.16 is 10 times greater than the given number. Therefore the given number is 20.16 ÷ 10 = 2.016. If we decrease the number 10 times we would get 2.016 ÷ 10 = 0.2016.
3	**C**	The smallest natural number which, when divided by 9 leaves a remainder of 6, is 6. Therefore 6 ÷ 4 = 1 (remainder of 2).
4	**C**	The sum of 18 and 12 is equal to the doubled speed of the boat in still water. Therefore the speed of the boat in still water is equal to 30 ÷ 2 = 15 km/h.
5	**A**	$=\frac{4}{2\times 4}-\frac{2}{2\times 4}+\frac{6}{4\times 6}-\frac{4}{4\times 6}+\frac{8}{6\times 8}-\frac{6}{6\times 8}=\frac{1}{2}-\frac{1}{4}+\frac{1}{4}-\frac{1}{6}+\frac{1}{6}-\frac{1}{8}$ $=\frac{1}{2}-\frac{1}{8}=0.375$
6	**B**	The natural numbers from 1 to 2016 are 2016. 1008 of them are even, and 202 end in the 5 - numbers 1.5, 3.5, 5.5, 7.5, 401.5, 403.5. The numbers which are divisible either by 2, or by 5, are 1210 in total. The numbers which are NOT divisible neither by 2, nor by 5, are 806.
7	**B**	The number of all proper fractions with one-digit denominators is 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36. The reducible fractions among them are: $\frac{2}{4},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{2}{8},\frac{4}{8},\frac{6}{8},\frac{3}{9},\frac{6}{9}$. The number of irreducible fractions is 36 – 9 = 27 in total.
8	**A**	Let us compare the sum of the numbers from the first column to the sum of the numbers from the second row:

<table>
<tr>
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<table>
<tr><td>$\frac{2}{3}$</td><td></td><td></td></tr>
<tr><td>Y</td><td>$1\frac{2}{3}$</td><td>X</td></tr>
<tr><td>$1\frac{1}{3}$</td><td></td><td></td></tr>
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$\frac{2}{3} + 1\frac{1}{3} = 1\frac{2}{3} + X \Rightarrow X = \frac{1}{3}$.
<table>
<tr><td>$\frac{2}{3}$</td><td>$2\frac{1}{3}$</td><td>2</td></tr>
<tr><td>3</td><td>$1\frac{2}{3}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>$1\frac{1}{3}$</td><td>1</td><td>$2\frac{2}{3}$</td></tr>
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<tr>
<td>9</td>
<td>A</td>
<td>The number would be divisible by 5 if $a = 0$ or $a = 5$.
The number would be divisible by 3 if $a = 0$
$1 + 2 + a + 3 + 4 + a + 5 + 6 + a + 7 + 8 + a = 36 + 4a$
That is only possible if $a = 0$.</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>The sides of each of the five identical rectangles are 1.5 cm and $1.5 \div 3 = 0.5\ cm$. Then $AB = 0.5 + 1.5 + 0.5 = 2.5\ cm$ and the area of the rectangle $ABCD = 1.5 \times 2.5 = 3.75\ sq.cm$</td>
</tr>
<tr>
<td>11</td>
<td>205</td>
<td>The last digit is 5, because the sum of 1, 2, 3, 4,...8 and 9 is 45.
The digit before last would be the last digit of the sum 1+2+3+...+7+8 plus 4, i.e. 36 + 4 = 40. The digit before last is 0.
The other digit we are looking for is equal to the last digit of 1+2+3+...+7 plus 4, i.e. 28 + 4 = 32. The last three digits are 2, 0 and 5. $\overline{abc} = 205$</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>The number of all pieces of all five chocolates is 5×28=140.
Therefore each child should get 140÷7 = 20 pieces.
By breaking one chocolate, we can get 20 pieces and have 8 left.
In this way we can give 20 pieces to 5 children, however there would be 2 more children and 5 more parts, each consisting of 8 pieces, left.
We can give 2 parts with 8 pieces each to each of the two children, and the fifth part, which consists of 8 pieces, we can divide in 2 parts of 4 pieces.
The number of times we would need to break the chocolates is 5 + 1 = 6.</td>
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<table>
<tr><td>13</td><td>6</td><td>The number B is a simple number and is a divisor of 9. Therefore $B = 3$ and $A = 9$, $A + B = 12$, and the natural numbers which are divisors of 12 are 6: 1, 2, 3, 4, 6 and 12.</td></tr>
<tr><td>14</td><td>5</td><td>The numbers smaller than 2016 are 6: 1026, 1062, 1206, 1260, 1602, 1620;
The numbers greater than 2016 are 11: 2061, 2106, 2160, 2601, 2610, 6012, 6021,6102, 6120, 6201, 6210.</td></tr>
<tr><td>15</td><td>3</td><td>The possible options are:
4 and 5, product 20;
6 and 7, product 42;
8 and 9, product 72.</td></tr>
<tr><td>16</td><td>2</td><td><table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>A</td><td></td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>B</td><td>+</td><td></td><td>–</td><td>+</td></tr>
<tr><td>C</td><td>+</td><td>–</td><td></td><td>–</td></tr>
<tr><td>D</td><td>+</td><td>+</td><td>–</td><td></td></tr>
</table>
If we add the number of hand shakes, the number must be divisible by 2, because each hand shake is counted twice.
In this case the number of hand shakes is $6 + x$.
We can mark the number of David’s handshakes with x. The number x can NOT be greater than 3.
$6 + x$ can be divided by 2 only if x is either 0 or 2.
However, x is not 0, because Adam shook hands with all the children. Therefore $x = 2$. David shook hands with 2 children.</td></tr>
<tr><td>17</td><td>4</td><td>Let us subtract 12 gold medalists from the total number of medal winners. Therefore 18 people have won bronze and silver medals.
The bronze medalists who have no gold medals are $13 – 5 = 8$.
The silver medalists are 14. We get $14 + 8 = 22$ in total.
$22 – 18 = 4$. Four athletes have won both bronze and silver medals.</td></tr>
</table>

18	**20**	The beads with numbers from 6 to 14 are situated between the beads with numbers from 5 to 15. The beads are 9 in total. The beads on the opposite side are also 9. If we also note the 2 beads numbered 5 and 15 we will find that the beads on Annie's necklace are $2 \times 9 + 2 = 20$.
19	**2**	In order for the value of the expression to be increased by 1, the following needs to be true: The first number in the expression $3 + 12 - 10$ needs to be exchanged with 4. Then the initial value would be increased by 1. $\square \div 2 = 4$ would be possible if we exchange 6 for 8. $6 \div \square = 4$ is not possible. The second number in the expression $3 + 12 - 10$ needs to be exchanged with 13. Then the initial value of the expression would be increased by 1. $\square \times 3 = 13$ is not possible. $4 \times \square = 13$ is also not possible. The third number in the expression 3+12-10 needs to be exchanged with 9. Then the initial value would be increased by 1. $1 \times \square = 9$, if we exchange 10 for 9. $\square \times 10 = 9$ is not possible. We can exchange two of the numbers in the expression, so that the initial value would be increased by 1: $8 \div 2 + 4 \times 3 - 1 \times 10$ $6 \div 2 + 4 \times 3 - 1 \times 9$
20	**2**	We would have to place 3 coins on each pan of the scales. **If the scales are balanced**, then we would have to place 1 of the remaining coins on each pan of the scales and if the scales are still balanced, the third remaining coin is fake; if it is not balanced, the lighter (fake) coin can be found on the higher pan of the scales. **If the scales are not balanced**, the lighter coin can be found on the higher pan of the scales. We would then have to compare the weights of two of those three coins.

FINAL 2016: GROUP 5

Problem 1. Find the sum of the fractions $\frac{1}{3}$ and $\frac{\square}{4}$, if $\frac{\square}{4} < \frac{1}{3}$.

A) $\frac{1}{3}$ **B**) $\frac{7}{12}$ **C**) $\frac{1}{4}$ **D**) $\frac{1}{3}$ or $\frac{7}{12}$

Problem 2. The numbers from 0 to 40 are written down one after another: 01234567891011...37383940.

In how many ways can we pick out two consecutive digits, so that their sum would be 10?

A) 4 **B**) 5 **C**) 6 **D**) 7

Problem 3. There are several points along a straight line. A student placed a point between every two adjacent points. After doing this a number of times, there were now 129 points along the straight line. How many points were possibly on the straight line originally (before the student placed any extra points)?

A) 2 **B**) 3 **C**) 4 **D**) 6

Problem 4. Adam has 44 marbles - blue, red, white and yellow. The number of the blue marbles is 2 more than that of the red, the number of the red marbles is 4 more than that of the white, and the number of the white marbles is 6 more than that of the yellow. How many blue marbles does Adam have?

A) 12 **B**) 14 **C**) 16 **D**) 18

Problem 5. Iva arrived at the bus stop and looked at her watch, which showed 08:01h. It meant that she was 2 minutes late for her bus. What she did not know was that her watch was running 5 minutes ahead. If the bus came 1 minute late, for how many minutes would Iva have to wait at the bus stop?

A) 4 **B**) 5 **C**) 3 **D**) 6

Problem 6. How many four-digit numbers are there that can be written down using the four digits 1, 2, 3 and 4, in such a way that 1 is not the digit of ones, 2 is not the digit of tens, 3 is not the digit of hundreds, and 4 is not the digit of thousands?

A) 6 **B**) 9 **C**) 12 **D**) 18

Problem 7. What is the number in which the digit of tenths is smaller than the digit of tens?

A) 222.31 **B**) 209.09 **C**) 32.32 **D**) 345.255

Problem 8. A storage room can be filled up with either 12 chests, or with 18 boxes. There are currently 4 chests and 9 boxes in the room. How many more boxes can fit into the room?

A) 6 **B**) 4 **C**) 2 **D**) 3

Problem 9. A spring with a flow rate of 84 *liters* of water per minute provides water for three fountains. Four times more water reaches the second fountain than does the first one, and half as much water reaches the third fountain than does the second one. How many *liters* per minute is the flow rate of the fountain which receives the least amount of water?

A) 4 **B**) 7 **C**) 12 **D**) 14

Problem 10. Which of the following fractions is equal to $\frac{4095}{6426}$?

A) $\frac{65}{102}$ **B**) $\frac{102}{65}$ **C**) $\frac{75}{112}$ **D**) $\frac{112}{75}$

Problem 11. I added each two of the numbers *A*, *B* and *C*, and then added the sums again. At last I got $2\frac{2}{5}$. What is $A + B + C$ equal to?

Problem 12. If the dividend is $\frac{1}{9}$, and the divisor is $\frac{1}{111111111}$, then the quotient is a number that is written down using *X* different digits. Calculate *X*.

Problem 13. Alex has 3 of each of the following coins: 1, 2, 5, 10, 20 and 50 cents. He wants to buy a book that costs 3 euros and 96 cents but he does not have enough money, so he asks his father for help. What fraction of the book price does his father need to pay? Write down the answer in the form of an irreducible fraction.

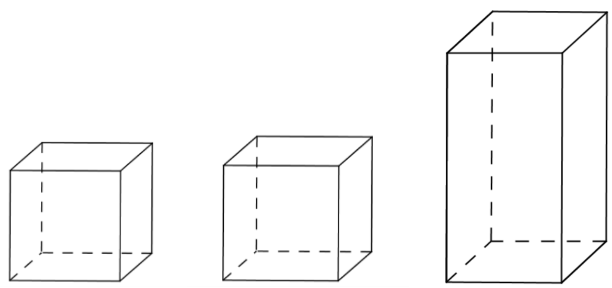
Problem 14. Ten teams are participating in a football tournament. Each team plays every other team exactly once. For each match, the winning team gets 3 points, the losing team gets 0 points, and in case of a draw each team gets 1 point. At some point in the tournament it turns out that the teams have earned a total of 131 points. How many games are yet to be played?

Problem 15. The two-digit numbers $\overline{ab}$, $\overline{bc}$ and $\overline{cd}$ are multiples of 17 (each letter represents one number). Calculate the greatest possible value of

$$a + b + c + d.$$

Problem 16. How many of the four-digit numbers written down using the four digits 2, 0, 1, 6 are divisible by 36?

Problem 17. A cuboid has been formed from two identical cubes with a surface area of 1.5 *sq. cm* each. Calculate the surface area of this cuboid.



Problem 18. The fraction $\frac{1}{14}$ is presented as an infinite repeating decimal. What are the digits that are **not** used when writing it down?

Problem 19. How many prime numbers x are there, smaller than 99, for which $2x + 1$ is also a prime number smaller than 99?

Problem 20. The numbers A and B are such that $143 \times A + 325 \times B = 6.5$. Calculate the value of the following expression if A and B are the same as above:

$$11 \times A + 25 \times B.$$

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**D**	$\frac{\square}{4} < \frac{1}{3} \Rightarrow \square = 0$ or $\square = 1$.
2	**C**	012345678910111213141516171819202122232425262728293031323334353637383940 The digit couples are: 91, 19, 28, 82, 37, 73.
3	**B**	If the number of the points is 3, we will place another 2; now there are 5 points and then we will place 4 more points to get 9. Then we will place another 8 points to get 17. Then we will place another 16 to get 33. Then we will place another 32 to get 65. Then we will place another 64 and the final number of the points will be 129.
4	**C**	If the yellow marbles are 4, then the white marbles are 10, the red are 14, and the blue are 16. The total number is 4 + 10 + 14 + 16 = 44.
5	**A**	The watch showed that the time was 08:01 h. She was 2 minutes late, which meant that she was supposed to arrive at 07:59 h, according to her watch. This meant that she came 3 minutes early because the bus (according to her watch) was supposed to arrive at 08:04 h. The bus was running 1 minute late. Therefore it would arrive at 08:05 h. Iva would have to wait for 4 minutes.
6	**B**	We can choose either 1, 2 or 3 for the digit of thousands. 1, 2 or 4 for the digit of hundreds; 1, 3 or 4 for the digit of tens and 2, 3 or 4 for the digit of ones. If the digit of thousands is 1, then the numbers are 1432, 1234 and 1243. If the digit of thousands is 2, then the numbers are 2134, 2143 and 2413.

		If the digit of thousands is 3, then the numbers are 3412, 3214 and 3142. The number of four-digit numbers are 9 in total.
7	**D**	In the number 345.255, the digit of tenths is 2 and is smaller than the digit of tens, which is 4.
8	**D**	$\frac{4}{12}+\frac{9}{18}=\frac{5}{6}$ of the room is occupied. The unoccupied part is equal to $\frac{1}{6}$, which means there is still space for 3 boxes.
9	**C**	We can carry out a check using the possible answers. If the flow rate of the second fountain is 48 *liters* per minute, then the flow rates of the first and third fountains would respectively be 12 *liters* per minute and 24 *liters* per minute. In this case 48 + 12 + 24 = 84.
10	**A**	The greatest common divisor of 4095 and 6426 is 63. After cancelling the fraction, we would get $\frac{65}{102}$ as a result.
11	$1\frac{1}{5}$ or 1.2	$2 \times A + 2 \times B + 2 \times C = 2\frac{2}{5} \Rightarrow A + B + C = 1\frac{1}{5}$.
12	8	The quotient is 12 345 679.
13	1/3	Alex is able to pay 3 × (1 + 2 + 5 + 10 + 20 + 50) = 264 cents. 3.96 – 2.64 = 1.32, so his father needs to pay 132/396=1/3 of the book price.
14	0 or 1	The total number of matches is 45 and the maximum points a team can earn is 135. Now since 131 points have been earned, there are 4 points left. Therefore 131 points can be earned in three ways: 41 wins and 4 draws, with no matches to go; 43 wins and 1 draw, with 1 more match to go; 43 wins, 1 draw and 1 defeat, with no matches to go.

15	21	The number $\overline{ab}$ can be 17, 34, 51, 68 or 85. The number $\overline{bc}$ can be 17, 34, 51, 68 or 85. We must keep in mind that the second digit of $\overline{ab}$ is also the first digit of $\overline{bc}$. Therefore the options are either $a = 6$ or $a = 8$. 1. $a = 6 \Rightarrow b = 8, c = 5, d = 1 \Rightarrow a + b + c + d = 20$; 2. $a = 8 \Rightarrow b = 5, c = 1, d = 7 \Rightarrow a + b + c + d = 21$. The sum we are looking for is 21.
16	6	The sum of the digits is 9, so all of the four-digit numbers written down using these digits are divisible by 9. In order for it to be divisible by 36, it also needs to be divisible by 4. In this case the last two digits must be arranged as follows: xx60, xx20, xx16 and xx12. It is now possible to find the four-digit numbers we are looking for: 1260, 2160, 6120, 1620, 2016 and 6012.
17	2.5	The length of the edge of the cube is 0.5 *cm*. We can find the surface area of the cuboid by subtracting the doubled area of one of the sides of the cube from the sum of the surface areas of the two cubes: 2 × 1.5 – 2 × 0.5 × 0.5 = 3 – 0.5 = 2.5 *sq. cm.*
18	3, 6 and 9	$\frac{1}{14} = 0.0\overline{714285}$ The digits 3, 6 and 9 are not used to write down the repeating decimal.
19	7	(2; 5), (3; 7), (5; 11), (11; 23), (23; 47), (29; 59), (41; 83).
20	0.5	$143 \times A + 325 \times B = 13 \times (11 \times A + 25 \times B) \Rightarrow$ $11 \times A + 25 \times B = 6.5 \div 13 = 0.5$.

TEAM COMPETITION – NESSEBAR, BULGARIA
MATHEMATICAL RELAY RACE

The answers to each problem are hidden behind the symbols **@, #, &, §** and ***** and are used in solving the following problem. Each team, consisting of three students of the same age group, must solve the problems in 45 minutes and then fill a common answer sheet.

GROUP 5

Problem 1. Subtract the number **@** from the numerator and from the denominator of the fraction $\frac{31}{67}$, in order to get a fraction equal to $\frac{1}{3}$. Calculate **@**.

Problem 2. We have made a random selection of **#** three-digit numbers. Among all three-digit numbers, there would always be at least 3 which are co-prime with **@**. Find the smallest possible value of **#**.

Problem 3. The children from a school class had to solve **#** problems for homework. Three of them solved respectively 60, 50 and 40 problems. At least **&** problems have been solved by all three of them. Find **&**.

Problem 4. Two ants started walking towards each other simultaneously from the two points *A* and *B*. One of the ants will travel the distance in **&** hours and the other one will travel the distance two hours faster. What part of the full distance would they need to walk before they meet, if two hours have passed since their departure? Denote the answer using **§**. Find **§**.

Problem 5. If $\S = \frac{1}{*+1} - \frac{1}{*+2}$, where ***** is an integer, find *****.

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	@ = 13	$\frac{31-13}{67-13} = \frac{18}{54} = \frac{1}{3}$
2	# = 72	The numbers which are not coprimes to 13 are **69.** Those numbers are: 13 × 8 = 104, 13 × 9 = 117, 13 × 10 = 130,..., 13 × 76 = 988. The number we are looking for in this case is **69** + 3 = 72.
3	**&** = 6	The first student did not solve 12 problems, the second did not solve 22, and the third did not solve 32 problems. If none of the other two students solved the unsolved problems, then the unsolved problems would be 66 in the worst case scenario. In this case 72 – 66 = 6 problems have been solved by the three of them.
4	§ = $\frac{1}{6}$	They have walked $\frac{2}{6} + \frac{2}{4} = \frac{10}{12}$ of the full distance. The remaining distance they have yet to walk is $1 - \frac{10}{12} = \frac{1}{6}$.
5	* = 1	$\frac{1}{6} = \frac{1}{2 \times 3} = \frac{3-2}{2 \times 3} = \frac{1}{2} - \frac{1}{3}$.