MATHEMATICS WITHOUT BORDERS

2015-2016

AUTUMN 2015: GROUP 4

Problem 1. What is the largest 4-digit number with 0 as a units digit?

A) 9,909 **B)** 9,990 **C)** 9,099

Problem 2. How many are all the possible digits that can be placed instead of @, so that $2015 < 2@15$ would be true?

A) 0 **B)** 8 **C)** 9

Problem 3. I thought of a number. I added it to 222 and got 1,000. The number I thought of is:

A) 1,222 **B)** 888 **C)** 778

Problem 4. How many are the 5-digit numbers that do NOT have 9 as a units digit?

A) 810 **B)** 8,100 **C)** 81,000

Problem 5. How many sheets of paper are there between the 3^{rd} and the 101^{st} pages of a book?

A) 99 B) 98 C) 48

Задача 6. Fill in the missing number in the box.

$$\square \div 20 + 15 - 2{,}015 = 0$$

A) 400 **B)** 4,000 **C)** 40,000

Problem 7. Find the value of $2{,}015 - 1 + 3 - 4 + 6 - 7 + 9 - 10 + 12 - 13 + 15$.

A) 2,025 **B)** 2,020 **C)** 2,015

Problem 8. There were 1,001 pieces of paper. Some of them were cut into three parts. Altogether, there are now 2,015 pieces of paper. How many pieces were cut into three parts?

A) 507 **B)** 494 **C)** 1,014

Problem 9. If the difference is 9,999 and the subtrahend is 1, the minuend is:

A) 9,998 **B)** 10,000 **C)** 1,000

Problem 10. A student only marked the odd numbered pages of his notebook by using only odd numbers such as 1, 3, 5 etc. He used 93 digits. How many pages does the notebook have?

A) 96 **B)** 97 **C)** 98

Problem 11. I chose a number and added it to 1. Then I multiplied the resulting sum by 2. After that I divided the resulting product by 3. What is the number I chose, if the quotient is 4, and the remainder is 2?

Problem 12. The three following actions have been applied to the number 2 in a random order:
- multiplying by 2
- dividing by 2
- adding 2

How many possible results are there?

Problem 13. There are less than 100 apples in a basket. These apples can be divided equally between 2, 3, or 5 children. These apples can NOT be divided equally between 7 children - one more apple would be needed to do that. What is the number of apples in the basket?

Problem 14. In $A + \overline{AB} = \overline{CDE}$ each letter corresponds to a digit. Identical letters correspond to identical digits and different letters correspond to different numbers. What is the greatest possible number that corresponds to $\overline{ABCDE}$?

Problem 15. How many missing addends are there in the expression
$3 + 6 + 9 + 12 + 15 + 18 + 21 + \cdots + 93 + 96 + 99$?

Problem 16. Which of the following is the smallest number: 62 345, 523 420 and 432 100?

Problem 17. How many pairs of integers which product is 63 can be selected from 0 to 99?

Problem 18. What is the next number in the sequence of numbers?
2, 11, 20, 101, 110, 200, 1001, 1010, 1100, 2000, 10 001, ...

Problem 19. In the number *A* the positions of the tens and hundreds were exchanged and the resulting number was 1,234. What is the number *A*?

Problem 20. How many seconds do we have to take out of 600 seconds to get 6 minutes?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**B) 9,990**	*,**0 ⇒9,990
2	**C) 9**	2015 < 2115, 2015 < 2215,…, 2015 < 2915
3	**C) 778**	1000 − 222 = 778
4	**C) 81,000**	The numbers are $9 \times 10 \times 10 \times 10 \times 9 = 81{,}000$
5	**C) 48**	These are the sheets s of paper with page numbers (5; 6), …, (99; 100). Their total number is 48.
6	**C) 40,000**	$((0 + 2{,}015) - 15) \times 20 = 2{,}000 \times\ 20 = 40{,}000$
7	**A) 2,025**	$2{,}015\ - 1 + 3 - 4 + 6 - 7 + 9 - 10 + 12 - 13 + 15 =$ $= (15 - 13) + (12 - 10) + (9 - 7) + (6 - 4) + (3 - 1) + 2{,}015 =$ $= 2{,}025.$
8	**A) 507**	The number of pieces increases by a number that is the doubled number of the pieces that have been cut. The number of pieces has increased by 1014. Therefore the number of pieces that have been cut is 507. Another way to find the answer is to check the given answers $(1{,}001 - 507) + 3 \times 505 = 494 + 1521 = 2{,}015.$
9	**B) 10,000**	9,999+1=10,000
10	**C) 98**	The odd one digit numbers are 5. Five digits have been used for them. The odd two digit numbers are 45. 90 digits have been used for them. In order to number the odd pages from 1 to 99, we must use 95 digits. However, only 93 digits have been used. Therefore the last odd page is page 97. The notebook has 98 pages.

11	**6**	Let us solve the problem starting at the end: the number which we divided by 3 is 4 × 3 + 2 = 14. The number that we multiplied by 2 is 14 ÷ 2 = 7. The number that we added to 1 is 7 – 1 = 6.
12	**3**	If we mark the actions with M, D and A, there are 6 ways for their sequential use: MDS, MSD, DMS, DSM, SMD, SDM. For each of these ways we obtain the following results: 4, 3, 4, 6, 4, 4. Thus the number of different results is 3: 3, 4 and 6.
13	**90**	The numbers divisible by 5 and by 2 with a remainder of 0 are: 10, 20, 30, 40, 50, 60, 70, 80, 90. The ones divisible by 3 are 30, 60 and 90. From the numbers 30+1, 60+1 and 90+1 only 91 is divisible by 7. There are 90 apples in the basket.
14	**98,107**	The sum $A + \overline{AB}$ is at most $9 + \overline{9B} = 9 + 98 = 107$. Therefore $\overline{ABCDE} = 98{,}107$.
15	**23**	The addends are the one digit (except 0) and two digit numbers, divisible by 3. The first number is 3, and the 33rd number is 99. The numbers that have been skipped are 33 – (7 + 3) = 23.
16	**62,345**	62,345 < 432,100 < 523,420
17	**3**	The numbers are 1 and 63; 3 and 21; 7 and 9.
18	**10,010**	These are the numbers with 2 as the sum of their digits. They have been arranged from smallest to biggest. The next number is 10,010.
19	**1,324**	1,234 ⇒1,324
20	**240**	600 – ☐ = 360

WINTER 2016: GROUP 4

Problem 1. What is the missing number?

$$? + 110 = 1{,}099 + 11$$

A) 1,000 **B)** 1,010 **C)** 990

Problem 2. Which three numbers are smaller than 30,020?

A) 30,019; 30,020; 30,021 **B)** 30,001; 30,010; 30,019 **C)** 30,001; 30,010; 31,000

Problem 3. If the difference is 24,345, and the subtrahend is 6,707, the minuend is:

A) 31,052 **B)** 17,638 **C)** 17,648

Problem 4. How many of the following expressions are correct?

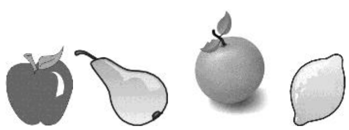
$$165 + 561 = 727$$

$$264 \times 5 - 2 = 264 \times 3$$

$$90{,}000 \div 10 < 10{,}000$$

A) 1 **B)** 2 **C)** 3

Problem 5. We have an apple, a pear, an orange and a lemon.



We must distribute them among two children.

In how many different ways can we distribute the fruit, so that each child would get 2 fruit?

A) 8 **B)** 6 **C)** 4

Problem 6. What is the hundreds digit of the smallest 5-digit number, that has a sum of its digits equal to 25?

A) 6 **B)** 9 **C)** 1

Problem 7. Which of the following numbers has 3 as a digit of hundreds and 1 as a digit of thousands?

A) 1,313 **B)** 3,311 **C)** 3,113

Problem 8. Last year 33 white, red and yellow tullips blossomed at the same time in Maya's garden. The white and red tullips together were 19, and the red and yellow tullips together were 18. Which tullips were of the greatest number?

A) red **B)** yellow **C)** white

Problem 9. The four-digit numbers smaller than 2,015 are:

A) 2,014 **B)** 1,015 **C)** 1,016

Problem 10. What digit should we place instead of *, so that the following sum would be correct?

$$\begin{array}{r} *1 \\ +*96 \\ \underline{92*} \\ 2016 \end{array}$$

A) 9 **B)** 8 **C)** 7

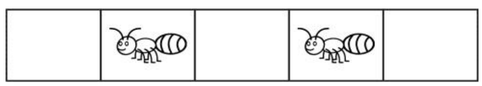
Problem 11. The natural numbers *A, B, C* and *D* are such that $A \times B = 2$, $B \times C = 6$ and $C \times D = 3$. What is the number D?

Problem 12. Three friends weigh respectively 24, 30 and 42 kilograms. They want to cross a river by using a boat that can carry a maximum of 70 kg. At least how many times would this boat need to cross the river, so that all three of them would get to the opposite shore?

Problem 13. The numbers 1,001; 1,008; 1,015; 1,022; …; 2,016 are recorded according the following rule: we get each following number by adding 7 to the preceding number, until we reach the number 2,016. How many numbers are there in total?

(*Hint:* 1,008 = 1,001 + 1 × 7; 1,015 = 1,001 + 2 × 7; 1,022 = 1,001 + 3 × 7, …..)

Problem 14. In how many rectangles do we find just one ant?



Problem 15. Place the numbers 1, 2, 3 and 4 in the squares in a way that would result in the greatest possible product. What is the product?

□ x □ x □□

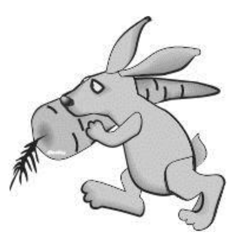
Problem 16. I have 5 baskets, each of which has 55 apples inside. If I move the apples to 11 baskets, and each basket has the same number of apples, how many apples would there be in each basket?

Problem 17. What is the digit of ones of the product of all odd one-digit numbers?

Problem 18. When *A* is divided by 5 the remainder is 2. What is the remainder when 3*A* is divided by 5?

Problem 19. What is the missing number? $\square \times 2 = 330 \div 5 + 330$

Problem 20. Bugs Bunny decided to eat only carrots for the week. He ate a different number of carrots each day, but he didn't eat more than 7 on any day. How many carrots did Bugs Bunny eat during that week?



ANSWERS AND SHORT SOLUTIONS

<table>
<tr><th>Problem</th><th>Answer</th><th>Solution</th></tr>
<tr><td>1</td><td>A</td><td>? + 110 = 1,110, i.e. ? = 1,000</td></tr>
<tr><td>2</td><td>B</td><td>30,001; 30,010; 30,019</td></tr>
<tr><td>3</td><td>A</td><td>? – 6,707 = 24,345
24,345 + 6,707 = 31,052</td></tr>
<tr><td>4</td><td>A</td><td>165 + 561 = 726, i.e. the first expression is wrong.
264 × 5 – 2 = 1318; 264 × 3 = 792, i.e. the second expression is wrong.
90,000 ÷ 10 = 9,000 < 10 000, i.e. the third expression is correct.
Of the three expressions, only the third one is correct.</td></tr>
<tr><td>5</td><td>B</td><td>There are 6 possibilities. If we number the fruit as 1, 2, 3 and 4, then they would be distributed as follows:
<table>
<tr><td>First child</td><td>Second child</td></tr>
<tr><td>1, 2</td><td>3, 4</td></tr>
<tr><td>1, 3</td><td>2, 4</td></tr>
<tr><td>1, 4</td><td>2, 3</td></tr>
<tr><td>2, 3</td><td>1, 4</td></tr>
<tr><td>2, 4</td><td>1, 3</td></tr>
<tr><td>3, 4</td><td>1, 2</td></tr>
</table>
</td></tr>
<tr><td>6</td><td>A</td><td>25 = 9 + 9 + 6 + 0 + 1, therefore the smallest five-digit number with a sum of its digits equal to 25, is 10699.</td></tr>
<tr><td>7</td><td>A</td><td>In the number 1,313 the digit of ones is 3, the digit of tens is 1, the digit of hundreds is 3, and the digit of thousands is 1.</td></tr>
<tr><td>8</td><td>C</td><td>The sum of the numbers 19 and 18 is equal to the number of the 33 flowers</td></tr>
</table>

		plus the number of red tullips. Therefore, the number of red tullips is 19 + 18 – 33 = 4, the number of white tullips is 15, and the number of yellow tullips is 14. The white tullips are of the greatest number.
9	B	The numbers from 1 to 2,014 are 2,014. Among those numbers, the numbers from 1 to 999 are not four-digit numbers. Therefore the number we are looking for is 2,014 – 999 = 1,015.
10	A	We can solve the problem by carrying a check by using the possible answers. We can start as follows: 1 + 6 + * = …6, if * = 9.
11	1	If A × B = 2 ⇒ A = 1 or A = 2. If A = 1 ⇒ B = 2 ⇒ C = 3 ⇒ D = 1. If A = 2 ⇒ B = 1 ⇒ C = 6 ⇒ D could not be a natural number such that 6 × D = 3. Hence D = 1.
12	3	Let *C* denotes the heaviest of the three friends, *A* - the lightest one, and *B* - the third one. It would be impossible for all three of them to cross the river in one go, because 24 + 30 + 42 = 96 > 70. Therefore the boat would have to return at least once, and the smallest possible number of river crossings would be 3. Following is an example of a way in which all three friends can cross the river to the opposite shore: *C* stays on one of the shores, while *A* and *B* cross over to the opposite shore. *A* crosses back to the initial shore. *A* and *C* now cross to the opposite shore together.
13	146	2,016 = 1,001 + ☐ × 7. Therefore ☐ × 7 = 1,015, i.e. ☐ = 145.
14	8	If we mark the rectangles with A, A1, B, A2, C, the A1 and A2 rectangles would be the ones with ants in them. Therefore there is one ant in A1, AA1,

		AA1B, BA1, A2, BA2, BA2C, A2C each.
15	252	All possible products are: $1 \times 2 \times 43 = 86$; $1 \times 2 \times 34 = 68$; $1 \times 3 \times 24 = 72$; $1 \times 3 \times 42 = 126$; $1 \times 4 \times 23 = 92$; $1 \times 4 \times 32 = 128$; $2 \times 3 \times 14 = 84$; $2 \times 3 \times 41 = 246$; $2 \times 4 \times 13 = 104$; $2 \times 4 \times 31 = 248$; $3 \times 4 \times 12 = 144$; $3 \times 4 \times 21 = 252$ - the greatest possible product.
16	25	The number of apples is $5 \times 55 = 275$. From $275 \div 11 = 25$, it follows that there are 25 apples in each basket.
17	5	The number 5 is one of the multipliers, and none of the multipliers is an even number. Therefore the product must end in 5.
18	1	*First way:* The numbers that when divided by 5 leave a remainder of 2, are: 2, 7, 12, 17, 22,... The tripled numbers are: 6, 21, 36, 51, 66, ... When divided by 5, they all leave a remainder of 1. *Second way:* Since *Dividend* = *Divisor* × *Quotient* + *Remainder*, we could present the numbers that when divided by 5 leave a remainder of 2 as follows: 5 × Quotient + 2. The tripled numbers could be presented as follows: 15 × Quotient + 6. Therefore the remainder we are looking for would be equal to the remainder of dividing 6 by 5, i.e. the remainder = 1.
19	198	$\square \times 2 = 330 \div 5 + 330$, therefore $\square \times 2 = 396$, i.e. $\square = 198$
20	28	$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$

SPRING 2016: GROUP 4

Problem 1. If $6{,}565 \div 5 = 1{,}308 + \square$, then $\square$ =:

A) 5 **B**) 6 **C**) 7

Problem 2. $2{,}016 \times 8 - 2{,}016 \times 6 =$

A) 2,016 **B**) 4,032 **C**) other

Problem 3. A hippo eats 200 *kg* of grass every day, which is three times less than the kilograms of grass that an elephant eats in a week. How many days would it take for a hippo and an elephant to eat 2 *tons* of grass?

A) 7 **B**) 8 **C**) 9

Problem 4. What is the natural number that is greater than the number which is greater by 9 than 99,979 and smaller than the number which is 9 times greater than 11,110?

A) 99,988 **B**) 99,989 **C**) 99,990

Problem 5. The product of 4 natural numbers is 72. The sum of these numbers is 15 and neither of them is 2. In this case, which is the greatest among these numbers?

A) 6 **B**) 8 **C**) 9

Problem 6. We have 5 identical chocolate bars, each consisting of 28 pieces. We have to divide them equally between 7 children. What is the minimum number of times we need to break each chocolate bar in order to do this?

A) 6 **B**) 7 **C**) 8

Problem 7. How many different three-digit numbers $\overline{5**}$ can you get as a product of the number 29 and a two-digit number?

A) 2 **B**) 3 **C**) 4

Задача 8. A book has been numbered as follows: the pages of the first sheet have been numbered as 1 and 2; the pages of the second - as 3 and 4, and so on, until the pages of the last sheet, which have been numbered as 227 and 228. If I open the book at a random place, what is a possible product of the numbers of the two pages that I have opened the book at?

A) 9,900 **B**) 10,100 **C**) 90

Problem 9. In the same room we have 2 grandmothers, 4 mothers, 4 daughters and 2 granddaughters. What's the smallest number of people that there could be in the room?

A) 4 **B**) 6 **C**) 8

Problem 10. Find the last digit of the number that is equal to the sum of the following numbers:

1, 12, 123,1234, ..., 12345678 and 123456789.

A) 0 **B**) 6 **C**) 5

Problem 11. Four children met together: Adam, Bobby, Charley and Daniel. Adam shook hands with 3 of these children, Bobby shook hands with 2, and Charley shook hands with 1. How many of the children's hands did David shake?

Problem 12. A three-digit number consists of the digits 1, 2 and 3. The digit 1 is not the digit of hundreds, and the digit 3 is not next to the digit 2. What is the number?

Problem 13. The product of a few different one-digit numbers is a number that is divisible by 10 (the remainder is 0), but is not divisible by 20 (the remainder is not 0). Which even numbers could be among the multipliers?

Problem 14. I placed 1 addition sign and 1 multiplication sign between the digits of the number 2016. Example: $2 + 01 \times 6$ or $20 + 1 \times 6$. How many different numbers will I get after calculating the expressions?

Problem 15. Annie has a magical necklace. Each bead of the necklace is numbered (1, 2, 3, 4 and so on). If between the beads numbered as 5 and 15 there is the same number of beads, what is the total number of beads on Annie's necklace?

Problem 16. In the garden of Rose there are 1,232 roses not yet in bloom and 1,168 that are blooming. Every day 4 new roses bloom and the ones that are already blooming do not fade. How many days will it take for the blossoming and non-blossoming roses to be an equal number?

Problem 17. A container, when full of water, weighs 994 *kg* and when half full, it weighs as much as 4 empty containers. How many kilograms does this container weigh when it is empty?

Problem 18. The natural numbers from 10 to 30 have been written on different cards (one on each card). What is the smallest number of cards that we would need to take without looking in order to make sure that there would be at least 2 numbers divisible by 3?

Problem 19. What is the greatest possible number of different three-digit numbers that we can add and get a three-digit number as a result?

Problem 20. The expression we are going to use for this problem is $6 \div 2 + 4 \times 3 - 1 \times 10$. Exchange one of the numbers from the expression with such a number that the initial value of the expression would be increased by 1. How many of the numbers can be changed?

Problem	Answer	Solution
1	**A**	$6\,565 \div 5 = 1308 + \square \Rightarrow 1313 = 1308 + \square \Rightarrow \square = 5$
2	**B**	$2016 \times 8 - 2016 \times 6 = 2016 \times (8-6) = 2016 \times 2 = 4032$
3	**A**	In one week a hippo can eat 1400 kg of grass, and an elephant can eat 600 kg of grass. The amount they can eat in total is 2000 kg = 2 tons.
4	**B**	The number which is greater than 99,979 by 9 is 99,988, and the number which is 9 times greater than 11,110 is 99,990. The number we are looking for is 99,989.
5	**B**	15 = 8 + 3 + 3 + 1 = 6 + 6 + 2 + 1 = 9 + 2 + 2 + 2 Therefore the number we are looking for is 8.
6	**A**	The number of all pieces of all five chocolates is 5×28=140. Therefore each child should get 140÷7 = 20 pieces. By breaking one chocolate, we can get 20 pieces and have 8 left. In this way we can give 20 pieces to 5 children, however there would be 2 more children and 5 more parts, each consisting of 8 pieces, left. We can give 2 parts with 8 pieces each to each of the two children, and the fifth part, which consists of 8 pieces, we can divide in 2 parts of 4 pieces. The number of times we would need to break the chocolates is 5 + 1 = 6.
7	**B**	29×17=493 <5**, 29×18=522, 29×19=551, 29×20=580, 29×21=609>5**, therefore there are 3 options.
8	**B**	If I open the book, there will be two pages, both numbered. The smaller number will be even, and the greater will be odd. The numbers will be consecutive. $9{,}900 = 99 \times 100$, $10{,}100 = 101 \times 100$, $90 = 9 \times 10$, therefore I have opened the book at the pages numbered as 100 and 101. A possible product is 10,100.
9	**B**	In order for one of the women to be a grandmother, she would need to have a daughter, and a granddaughter. Therefore if there are two grandmothers, who are also mothers, they have one daughter each, i.e. 2 daughters, each of whom is also a mother to 1 granddaughter – 2 granddaughters, who are also daughters. The two granddaughters are also 2 daughters.

<table>
<tr><td></td><td></td><td>There are now 2 daughters left, who are also 2 mothers.
There are now 2 mothers left, who are also 2 grandmothers</td></tr>
<tr><td>10</td><td>C</td><td>The digit we are looking for is the last digit of the sum of the last digits, i.e. of 1 + 2 + 3 + ... + 8 + 9 = 45.</td></tr>
<tr><td>11</td><td>2</td><td><table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>A</td><td></td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>B</td><td>+</td><td></td><td>–</td><td>+</td></tr>
<tr><td>C</td><td>+</td><td>–</td><td></td><td>–</td></tr>
<tr><td>D</td><td>+</td><td>+</td><td>–</td><td></td></tr>
</table>
If we add the number of hand shakes, the number must be divisible by 2, because each hand shake is counted twice.
In this case the number of hand shakes is 6 + x.
We can mark the number of David’s handshakes with x. The number x can NOT be greater than 3.
6 + x can be divided by 2 only if x is either 0 or 2.
However, x is not 0, because Adam shook hands with all the children.
Therefore x = 2. David shook hands with 2 children.</td></tr>
<tr><td>12</td><td>312 or 213</td><td>The number is either *1* or **1.
If the number is *1*, then the number is either 312 or 213.
If the number is **1, then the digit 3 and the digit 2 are next to each other, which would mean that 1 is not the digit of ones.</td></tr>
<tr><td>13</td><td>2 or 6</td><td>The product of the one-digit odd numbers is a number which ends in 5. If we multiply it by 2 or by 6 we will get a number that ends in 0, but is not divisible by 20.</td></tr>
<tr><td>14</td><td>4</td><td>2×016 ⇒ 2×0+16=16; 2×01+6=8
20×16 ⇒2+0×16=2; 20×1+6=26
201×6 ⇒2 + 01 × 6 = 8; 20 + 1×6 = 26;</td></tr>
<tr><td>15</td><td>20</td><td>The beads with numbers from 6 to 14 are situated between the beads with numbers from 5 to 15. The beads are 9 in total. The beads on the opposite side are also 9. If we also note the 2 beads numbered 5 and 15 we will find that the beads on Annie’s necklace are 2×9+2=20.</td></tr>
<tr><td>16</td><td>8</td><td>The roses which are blooming and the roses which are not yet in bloom are 400 in total. The number of the roses in bloom needs to be increased</td></tr>
</table>

		by 32 roses. This will happen in 32÷3=8 days.
17	**142**	The water in a half full container weighs as much as 3 empty containers. The water in a full container weighs as much as 6 empty containers. The water in a full container (the weight of the water + the actual container) weighs as much as 7 empty containers. Therefore an empty container would weigh 994 ÷ 7 = 142 *kg*.
18	**16**	The numbers are $30 - 10 + 1 = 21$ in total. 7 of them are divisible by 3 with a remainder of 0, 7 are divisible by 3 with a remainder of 1, and 7 are divisible by 3 with a remainder of 2. We would need to take 7 + 7 + 2 = 16 cards, in order to make sure that we have taken 2 cards that have such numbers on them which when divided by 3 leave a remainder of 2.
19	**9**	101 + 103 + 105 + 107 + 109 + 111 + 113 + 115 + 117 = 981; 101 + 103 + 105 + 107 + 109 + 111 + 113 + 115 + 117 +119 = 1100.
20	**2**	In order for the value of the expression to be increased by 1, the following needs to be true: The first number in the expression 3 + 12 – 10 needs to be exchanged with 4. Then the initial value would be increased by 1. ☐÷2= 4 would be possible if we exchange 6 for 8. 6÷☐= 4 is not possible. The second number in the expression 3 + 12 – 10 needs to be exchanged with 13. Then the initial value of the expression would be increased by 1. ☐×3= 13 is not possible. 4×☐=13 is also not possible. The third number in the expression 3+12-10 needs to be exchanged with 9. Then the initial value would be increased by 1. 1×☐ = 9, if we exchange 10 for 9. ☐×10=9 is not possible. We can exchange two of the numbers in the expression, so that the initial value would be increased by 1: 8 ÷ 2 + 4 × 3 – 1 × 10 6 ÷ 2 + 4 × 3 – 1 × 9

FINAL 2016: GROUP 4

Problem 1. If $\overline{1234567\square} \times \square = 111111111$, then $\square$ = (   ).

A) 1 **B**) 7 **C**) 9

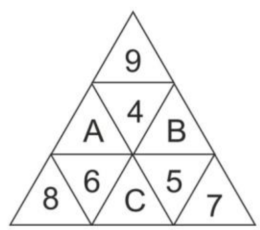
Problem 2. *A* is a four-digit number. Its digit of tens is two times greater than its digit of ones, its digit of hundreds is two times greater than its digit of tens. How many *A* numbers with this property are there?

A) 1 **B**) 9 **C**) 18

Problem 3. There are several points along a straight line. A student placed a point between every two adjacent points. After doing this 5 times, there were now 33 points along the straight line. How many points were originally on the straight line (before the student placed any extra points)?

A) 2 **B**) 3 **C**) 5

Problem 4. An equilateral triangle with a side length of 3 *cm* has been divided into 9 smaller equilateral triangles with a side length of 1 *cm*. The numbers *A, B, C,* 4, 5, 6, 7, 8 and 9 have been placed inside the smaller triangles. Now there are three equilateral triangles with sides of 2 *cm* on the diagram and the sums of the numbers inside them are equal.



What, then, is the greatest of those three numbers: *A, B* or *C*?

A) *A* **B**) *B* **C**) *C*

Problem 5. Iva arrived at the bus stop and looked at her watch, which showed 08:01h. It meant that she was 2 minutes late for her bus. What she did not know was that her watch was running 5 minutes ahead. If the bus came 1 minute late, for how many minutes would Iva have to wait at the bus stop?

A) 4 **B**) 5 **C**) more than 5

Problem 6. The number of astronomical hours in 2016 is equal to (   ).

A) $365 \times 12 \times 2$ **B**) $122 \times 9 \times 8$ **C**) $366 \times 30 \times 2$

Problem 7. A book is numbered as follows: the pages on the first sheet are numbered as 1 and 2, the pages on the second sheet are numbered as 3 and 4, and so on, until the last sheet, where the pages have been numbered as 127 and 128. If I tear off 11 consecutive sheets and add up all the page numbers of their 22 pages, which of the following sums is possible?

A) 255 **B**) 275 **C**) 341

Problem 8. A storage room can be filled up with either 12 chests, or with 18 boxes. There are currently 4 chests and 9 boxes in the room. How many more chests can fit into the room?

A) 6 **B**) 3 **C**) 2

Problem 9. A spring with a flow rate of 84 *liters* of water per minute provides water for three fountains. Four times more water reaches the second fountain than does the first, and half as much water reaches the third fountain than does the second. How many *liters* per minute is the flow rate of the fountain which receives the greatest amount of water?

A) 56 **B**) 48 **C**) 52

Problem 10. What is the three-digit number $\overline{abc}$, such that $\overline{abc} < 1116 \div 9,$ and can be presented as the product of both 4, and of 5 consecutive natural numbers?

A) 124 **B**) 120 **C**) 100

Problem 11. Find $\overline{ab}$ if

$2 + 24 + 246 + 2468 + 24680 + 246808 + 2468086 + 24680864 + 246808642 = \overline{\ldots ab}.$

Problem 12. We can move from square *A* to square *B* by moving either horizontally or vertically from one square to another. How many different routes that go through exactly 4 squares are there?

		B
A		

Problem 13. Alex and Boris both have 3 of each of the following coins: 1, 2, 5, 10, 20 and 50 cents. Boris uses 10 of his coins to create the smallest possible sum, and Alex uses 10 of his coins to create the greatest possible sum. How much smaller is Boris' sum than that of Alex?

Problem 14. Five people *(A*, *B*, *C*, *D* and *E)* are waiting in a queue. *C* is between *E* and *D*, *A* is next to *E*, and *B* is NOT the last. Which one is the last?

Problem 15. How many three-digit numbers $\overline{abc}$ are there, if 17 can divide both $\overline{ab}$ and $\overline{bc}$ without a remainder?

Problem 16. Seven numbers are arranged in a specific order and two of them are covered by shells. How many times is the number under the first shell smaller than the number under the second shell?

1,  **, 2, 6, 24,**  **, 720**

Problem 17. At least how many of the numbers do we need to change in order that the products of the numbers in the diagonals, rows and columns will be the same?

1	4	8
16	4	1
2	4	8

Problem 18. For a football game, the winner earns 3 points and the loser earns 0 points. If the match is drawn, each team gets 1 point. After 7 games, a team has earned 11 points. What is the possible number of games that the team has lost?

Problem 19. A number is *perfect* when the sum of its divisors (except the number itself) equals the given number. For example, the number 6 is called perfect because it is equal to the sum of 1 + 2 + 3, where 1, 2 and 3 are all its divisors except the number 6 itself. The next perfect number is an even number greater than 24 and smaller than 30. What is the number?

Problem 20. The expression we are going to use for this problem is 6 ÷ 2 + 4 × 3 – 1 × 10. Exchange one of the numbers from the expression with such a number that the initial value of the expression would be increased by 2. How many of the numbers can be changed?

Problem	Answer	Solution
1	**C**	$\overline{1234567\square} \times \square = 111111111$, so $\square \times \square$ would be a number digit of ones is 1. Therefore the possibilities are either 1 or 9. If we check both possibilities, we will find the correct answer: 9.
2	**C**	The numbers are one of two types: either $\overline{*421}$ or $\overline{*842}$. There are 9 possibilities for the digit of thousands for each of the two numbers. Therefore there are 18 numbers with such property.
3	**A**	If the number of points is 2, then we will place 1. There are now 3 points, we will place another 2. There are now 5 points, we place another 4. There are now 9, we place 8. There are now 17 points, we place 16. There are now 33 points.
4	**B**	Let us compare the sums: $14 + A + C = 12 + B + C = A + B + 13$. $14 + A + C = A + B + 13 \Rightarrow B = C + 1 \Rightarrow B > C$ $14 + A + C = 12 + B + C \Rightarrow B = A + 2 \Rightarrow B > A$ Answer: *B*.
5	**A**	The watch showed that the time was 08:01h. She was 2 minutes late, which means that she was supposed to arrive at 07:59h, according to her watch. This meant that she came 3 minutes early because the bus (according to her watch) was supposed to arrive at 8:04h. The bus was running 1 minute late. Therefore it would arrive at 8:05h. Iva would have to wait for 4 minutes.
6	**B**	2016 is a leap year, so $366 \times 24 = 122 \times 9 \times 8$.
7	**C**	If I tear off sheets 1-11: $\underbrace{1 + 2 + 3 + \ldots + 20 + 21 + 22}_{22\ addends} = 253$; If I tear off sheets 2-12: $\underbrace{3 + 4 + 5 + \ldots + 22 + 23 + 24}_{22\ addends} = 297$; If I tear off sheets 3-13: $\underbrace{5 + 6 + 7 + \ldots + 24 + 25 + 26}_{22\ addends} = 341$.
8	C	Half of the room is taken up by the boxes. There are already 4 chests in

<table>
<tr><td></td><td></td><td>the other half. It will take 6 chests to fill half of the space. Therefore there is room for another 2 chests.</td></tr>
<tr><td>9</td><td>B</td><td>We can carry a check using the possible answers. If the flow rate of the second fountain is 48 liters per minute, then the flow rates of the first and third fountains respectively would be 12 liters per minute and 24 liters per minute. In this case 48 + 12 + 24 = 84.</td></tr>
<tr><td>10</td><td>B</td><td>We are looking for a three-digit number smaller than 124.
The number is 120.
In fact $120 < 124$ and $120 = 1 \times 2 \times 3 \times 4 \times 5 = 2 \times 3 \times 4 \times 5$.</td></tr>
<tr><td>11</td><td>20</td><td>246808642
24680864
2468086
246808
24680
2468
246
24
2
Add all the digits of ones of these numbers: $2 \times (2 + 4 + 6 + 8) = 40$. 0 is the digit of ones we are looking for and 4 need to be added to the sum of all the digit of tens: $2 \times (4 + 6 + 8) + 2 + 4 = 42$, 2 is the digit of tens we are looking for.
Therefore $\overline{ab}$ is equal to 20.</td></tr>
<tr><td>12</td><td>10</td><td>$abcd, abxd, abxy, a\alpha xd, a\alpha xy, a\alpha\beta y,$
$\delta\alpha xd, \delta\alpha xy, \delta\alpha\beta y, \delta\varepsilon\beta y.$
<table><tr><td>c</td><td>d</td><td>B</td></tr><tr><td>b</td><td>x</td><td>y</td></tr><tr><td>a</td><td>α</td><td>β</td></tr><tr><td>A</td><td>δ</td><td>ε</td></tr></table></td></tr>
<tr><td>13</td><td>211</td><td>The smallest sum is $3 \times 1 + 3 \times 2 + 3 \times 5 + 10 = 34$, and the greatest sum is $3 \times 50 + 3 \times 20 + 3 \times 10 + 1 \times 5 = 245$.
245 – 34 = 211.</td></tr>
</table>

14	**A** or **D**	First we can arrange the 4 people as follows: *DCEA* or *AECD.* Then *B* may be situated as follows: *BDCEA* *DCEAB* *AECDB* *BAECD* Since *B* is not the last, *A* or *D* can be the last.
15	3	The number $\overline{ab}$ can be 17, 34, 51, 68 or 85. The number $\overline{bc}$ can be 17, 34, 51, 68 or 85. The second digit of $\overline{ab}$ is the first digit of $\overline{bc}$. Therefore the possibilities are the following: $b = 1, b = 5$ or $b = 8$. We get the numbers 517, 685 and 851.
16	120	Find the pattern first: after the first number 1, the rest are equal to the number before multiplying 1, 2, 3, 4, 5, 6 respectively, i.e.: 1; 1 × 1=1; 1 × 2 = 2; 2 × 3 = 6; 6 × 4 = 24; 24 × 5=120; 120 × 6 = 720. Therefore the numbers under the shells are 1 and 120. $120 \div 1 = 120$.
17	1	If we change the number 1 in the first row, first column, to 2, we will get a magic square.
18	0 or 2	11 points can be earned in the following ways: 2 wins, 5 draws and 0 defeats or 3 wins, 2 draws and 2 defeats.
19	28	The number is 28. 28 = 1 + 2 + 4 + 7 + 14.
20	2	$10 \div 2 + 4 \times 3 - 1 \times 10$ or $6 \div 2 + 4 \times 3 - 1 \times 8$.

TEAM COMPETITION – NESSEBAR, BULGARIA

MATHEMATICAL RELAY RACE

The answers to each problem are hidden behind the symbols **@, #, &, §** and ***** and are used in solving the following problem. Each team, consisting of three students of the same age group, must solve the problems in 45 minutes and then fill a common answer sheet.

GROUP 4

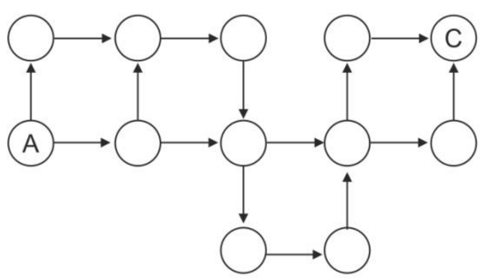
Problem 1. The maximum number of people that can attend a party is **@**, among whom there can not be two people born in the same month. Find **@**.

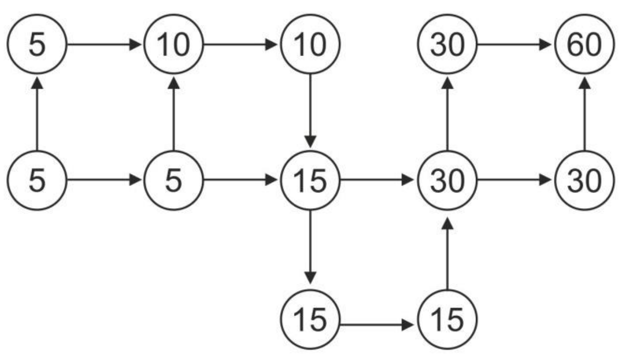
Problem 2. The sum of the digits, which are not part of the equation $123456 \div @ = \overline{\boldsymbol{abcde}}$, is #. Find **#**.

Problem 3. Use **&** to represent the greatest possible product of two integers with a sum of **#**. Find **&**.

Problem 4. There are 18 children in a class. Each of them has either 3 or 5 balloons. The total number of balloons is **&**. The number of children who have 5 balloons is **§**. Find **§.**

Problem 5. There are **§** ways to travel from point *X* to point *A*. There are * ways to travel from point *X* to point *C*. This must be done by going through point *A* and then following the arrows from point *A* to point *C*. Find *****.



Problem	Answer	Solution
1	**@ = 12**	If 13 people are present, there would definitely be 2 among them, who have been born in the same month. (Dirichlet's Principle)
2	**# =16**	123456 ÷ 12=1088 , therefore the missing digits are 7 and 9.
3	**& = 64**	16 = 0 + 16 = 1 + 15 = 2 + 14 = 3 + 13 = 4 + 12 = 5 + 11 = 6 + 10 = 7 + 9 = 8 + 8, therefore the possible products are 0, 15, 28, 39, 48, 55, 60, 63 and 64. The greatest is 64.
4	**§ = 5**	If all children are carrying 3 balloons each, then 3 × 18 = 54. Therefore there would be 64−54=10 balloons left. We would give those away to 5 children. In this way 13 children would be carrying 3 balloons each, and 5 children would be carrying 5 balloons each.
5	*** = 60**	 If there are *x* ways to reach the point *X*, and *y* ways to reach the point *Y*, then there would be *x*+*y* ways to reach the point *Z*.