MATHEMATICS WITHOUT BORDERS

2015-2016

AUTUMN 2015: GROUP 3

Problem 1. In how many ways can we place a digit instead of @ so that $139 > 1@0$ will be true?

A) 3 **B)** 4 **C)** 5

Problem 2. The unknown addend @ in the equation 3 *dm* = @ *dm* + 20 *cm* is:

A) 1 **B)** 10 **C)** 17

Problem 3. To what number do you add 13 to get 101?

A) 87 **B)** 88 **C)** 89

Problem 4. The number 3 is NOT the sum of:

A) 3 consecutive numbers **B)** 4 consecutive numbers **C)** 2 consecutive numbers

Problem 5. How many even numbers is the magic square made of?

6	8	1
	5	
	2	

Hint: *0 is an even number.*

A) 6 **B)** 7 **C)** 9

Problem 6. How many sheets of paper are there between the 23 and 77 pages of a book?

A) 25 **B**) 26 **C)** 55

Problem 7. From which number, if we subtract 11, we will get 89?

A) 80 **B)** 90 **C)** 100

Problem 8. In a box there are 30 pencils, of 3 different colours - 10 blue, 10 red and 10 green ones. What is the smallest possible number of pencils we would need to take out, without looking at their colour, in order to ensure that we have taken out pencils from all three colours?

A) 11 **B)** 21 **C)** 29

Задача 9. Fill in the missing number in the box.

$$\square \div 3 \times 2 + 15 - 25 = 0$$

A) 18 **B)** 15 **C)** 12

Problem 10. How many are the numbers between 12 and 120, which have at least two digits of 1?

A) 10 **B)** 11 **C)** 12

Problem 11. What is the number of possible *different* sums that we get when we add the results from throwing 4 dice?

Problem 12. Find the value of $20 + 1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + 17 - 19$.

Problem 13. There were 9 pieces of paper. Some of them were cut into three parts. Altogether, there are now 19 pieces of paper. How many pieces were cut into three parts?

Problem 14. A textbook is opened at random. To what pages is it opened if the sum of the facing pages is 89?

Problem 15. What are the last 2 digits of the sum

$$\underbrace{1 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 4 + 4 + \cdots + 9}_{45\ addends}.$$

Problem 16. How many numbers between 1 and 99 are divisible by 2 and 6?

Problem 17. It is known that :

- Among A, B , C and D there are two excellent students;
- Among A, B and C there is one excellent student;
- Among A, C and D there is one excellent student.

How many are the excellent students?

Problem 18. How many seconds do we have to take out of 72 seconds to get 1 minute?

Problem 19. Use 1, 2, 3, 4 and 5 to form a 2-digit number and a 3-digit number. Find the largest sum of these two numbers.

Problem 20. How many sticks with a length of 11 *cm* can we cut off from a stick with a length of 1 *m*?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**B) 4**	139>100; 139>110; 139>120; 139>130
2	**A) 1**	3 *dm* = 1 *dm* + 20 *cm*
3	**B) 88**	101 – 13 = 88
4	**B) 4**	3=1+2; 3=0+1+2
5	**A) 6**	<table><tr><td>6</td><td>8</td><td>1</td></tr><tr><td>0</td><td>5</td><td>10</td></tr><tr><td>9</td><td>2</td><td>4</td></tr></table>
6	**B) 26**	These are the lists of paper with page numbers (25,26),…, (75,76).
7	**C) 100**	100 – 11 = 89
8	**B) 21**	If the first 20 pencils are two of the colors, the 21st will be of the third color.
9	**B) 15**	((0+25) - 15) ÷ 2 × 3 = 15
10	**B) 11**	101,110, 111, …, 119
11	**21**	The smallest sum is 1+1+1+1=4, …, the greatest is 6+6+6+6=24. From 4 to 24 the possible sums are 21.
12	**10**	We subtract the number 2 from the number 20 five times.

13	**5**	If we cut 1 list of paper into 3 smaller lists, their number would be 8 + 3 = 11. If we cut 2 lists of paper into 3 smaller lists, their number would be 7 + 6 = 13. If we cut 3 lists of paper into 3 smaller lists, their number would be 6 + 9 = 15. If we cut 4 lists of paper into 3 smaller lists, their number would be 5 + 12 = 17. If we cut 5 lists of paper into 3 smaller lists, their number would be 4 + 15 = 19.
14	**44 and 45**	89 = 44 + 45
15	**85**	1+4+9+16+25+36+49+64+81=285
16	**16**	The numbers are 6, 12, 18, …, 90, 96.
17	**2**	If A is an excellent student, then from the second and third statement it follows that B, C and D cannot be excellent students. Therefore 1 of the statements is not true. A is not an excellent student. If B is an excellent student, then C cannot be an excellent student (as follows from the second statement). Therefore D must be an excellent student. If B is *not* an excellent student, then C and D would be excellent students (first statement). However in this case the third statement could not be true. Answer: The excellent students are B and D. There are two of them.
18	**12**	72 – 12 = 60 seconds = 1 minute
19	**573**	573= 542+31=541+32=531+42=532+41
20	**9**	$9 \times 11 = 99 < 100$ (см) = 1 (м)

WINTER 2016: GROUP 3

Problem 1. What is the missing number?

$$?-58=199+1$$

A) 142 **B)** 258 **C)** 242

Problem 2. The sum of $247+178+325$ is:

A) 750 **B)** 730 **C)** 650

Problem 3. What is the missing number?

$$\square \times 5=82-52$$

A) 6 **B)** 7 **C)** 8

Problem 4. How many of the following expressions are correct?

$$639-27\div 3 = 630$$
$$(478-458)\div 4 = 5$$
$$1000-319=681$$

A) 1 **B)** 2 **C)** 3

Problem 5. $(9\times 4)\div(3\times 2) =$

A) $9\times 4\div 3\times 2$ **B)** $(9\times 4)\div 3\times 2$ **C)** $9\times 4\div (3\times 2)$

Problem 6. What is the sum of the numbers in the 9th row?

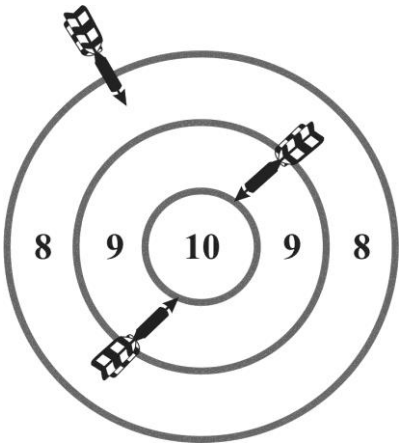
1	2	3
4	5	6
7	8	9
.....	.....	.....

A) 99 **B)** 78 **C)** other

Problem 7. There is a basket in a dark room. In the basket there are 6 yellow, 5 red and 4 green apples. What is the smallest possible number of apples we would need to take out, without looking at their colour, in order to ensure that we have taken out apples from all three colours?

A) 3 **B)** 12 **C)** 13

Problem 8. If a person participating in an archery contest hits the target in all 3 attempts, how many different results can they get? Keep in mind that the result is the sum of all 3 attempts. (If the arrow falls between 8 and 9, 9 points are counted. If it falls on the line between 9 and 10, 10 points are counted.)



A) 5 **B)** 6 **C)** 7

Problem 9. If we add the number equal to 300 + 100 to the number equal to 200 + 400, we would get

A) 100 **B)** 1,000 **C)** 1,100

Problem 10. A gallery has 360 paintings. 101 of them were sold on the first day. The paintings sold on the second day were 31 more than those sold on the first day. How many paintings are still not sold?

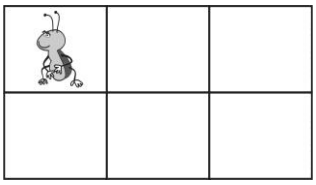
A) 132 **B)** 127 **C)** 228

Problem 11. Three friends weigh respectively 24, 30 and 42 kilograms. They want to cross a river by using a boat that can carry a maximum of 70 *kg*. At least how many times would this boat need to cross the river, so that all three of them would get to the opposite shore?

Problem 12. By how much is the larger sum greater than the smaller sum?

$$\begin{array}{r} 1 \\ +\ 12 \\ \hline 123 \end{array} \qquad \begin{array}{r} 321 \\ +\ 31 \\ \hline 3 \end{array}$$

Problem 13. In how many rectangles can we find the ant? (Keep in mind that a square is also a rectangle.)



Problem 14. Place the digits 1, 2, 7, 8 and 9 in the squares in such a way that after calculating, the result would be the greatest possible number. What is the number?

$$\square + \square + \square - \square\square$$

Problem 15. Boko and Tsoko went fishing with their sons. All of them caught an equal number of fish. How much fish did each of them catch, if they caught 9 fish in total?

Problem 16. What is the greatest possible sum of three odd one-digit numbers?

Problem 17. The numbers 1, 10, 19, 28,…, 82, 91 have been written down according to the following rule: we get each following number by adding 9 to the preceding number, until we reach 91. How many numbers have been written down?
(***Hint:*** $10 = 1 + 1 \times 9$; $19 = 1 + 2 \times 9$; $28 = 1 + 3 \times 9$, …)

Problem 18. I chose a number. I subtracted 555 from it and got 166 as a result. What was the number I chose?

Problem 19. What is the smallest three-digit number with 18 as the sum of its digits?

Problem 20. How many tens are there in the number that we get after calculating
$9 + 91 + 18 + 82 + 27 + 73 + 36 + 64 + 45 + 55$?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**B**	? – 58 = 200; ? = 200 + 58 = 258
2	**A**	247 + 178 + 325 = 425 + 325 = 750
3	**A**	☐ × 5 = 30; ☐ = 6
4	**C**	All three expressions are correct.
5	**C**	The value of the expression is 6. The possible answers are respectively 24, 24 and 6.
6	**B**	The numbers in the 9th row are 25, 26, and 27 (read from left to right). Their sum is 78.
7	**B**	In the worst case scenario we would first take out all 6 yellow apples, after that we would take out all 5 red apples, after which we would take out 1 green apple. We would then have taken out apples from all three colours. 6 + 5 + 1 = 12
8	**C**	The smallest result is 24, and the greatest is 30. There are 7 possible results in total: 24 = 8 + 8 + 8; 25 = 8 + 8 + 9; 26 = 8 + 9 + 9 = 8 + 8 + 10; 27 = 9 + 9 + 9 = 8 + 9 + 10; 28 = 9 + 9 + 10 = 8 + 10 + 10; 29 = 9 + 10 + 10; 30 = 10 + 10 + 10
9	**B**	(300 + 100) + (200 + 400) = 400 + 600 = 1000
10	**B**	The paintings sold on the second day were 101 + 31 = 132. The paintings sold on the first and second day together were 233. The paintings remaining unsold in the gallery are 360 – 233 = 127.
11	**3**	Let *C* denotes the heaviest of the three friends, *A* - the lightest one, and *B* - the third one. It would be impossible for all three of them to cross the river in one go, because 24 + 30 + 42 = 96 > 70. Therefore the boat would have to return at least once, and the smallest possible number of river crossings would be 3. Following is an example of a way in which all three friends can cross the river

		to the opposite shore: C stays on one of the shores, while A and B cross over to the opposite shore. A crosses back to the initial shore. A and C now cross to the opposite shore together.
12	**219**	1 321 + 12 + 31 123 3 The first sum is 136, and the second sum is 355. 355 – 136 = 219
13	**6**	The ant can be found in 1 square 1×1, in 2 rectangles 1×2, in 1 rectangle 1×3; in 1 square 2×2, in 1 rectangle 3×2.
14	**12**	7 + 8 + 9 – 12 = 12
15	**3 or 1**	If we assume that the problem speaks of four people – two fathers and two sons, then the result would be impossible, because 9 is not divisible by 4. Therefore the problem must speak of three people: a grandfather, his son, and his grandson, or of 9 people: two fathers and seven sons.
16	**27**	9 + 9 + 9 = 27
17	**11**	We get the second number by adding 9 once (1 + 1 × 9) to 1. We would get 91 by adding the number 9 ten times to the number 1 (1 + 10 × 9). Therefore the numbers that have been written down are 11.
18	**721**	555 + 166 = 721
19	**189**	The number is presented as 1*xy*, where $x + y = 17$. 17 is presented in two ways: as the sum of the two numbers 9 and 8, or 8 and 9. The numbers of the type 1*xy* are two: 189 and 198. The number we are looking for is 189.
20	**50**	9 + 91 + 18 + 82 + 27 + 73 + 36 + 64 + 45 + 55 = (9 + 91) + (18 + 82) + (27 + 73) + (36 + 64) + (45 + 55) = 100 + 100 + 100 + 100 + 100 = 500 In the number 500 there are 5 hundreds, 50 tens and 500 units.

SPRING 2016: GROUP 3

Problem 1. If $(4 + 7 + 9) \times 6 = 24 + 42 + \square$, then $\square$ = ?

A) 54 **B**) 48 **C**) 60

Problem 2. 1,000 – (12 + 23 + 34 + 45 + 55 + 66 + 77 + 88) = ?

A) 400 **B**) 500 **C**) other

Problem 3. One *kg* of dried mushrooms is derived from 12 *kg* of fresh mushrooms. How many *kg* of fresh mushrooms would you need to get 6 *kg* of dried mushrooms?

A) 2 **B**) 18 **C**) 72

Problem 4. Two ants are moving towards each other. One of them travelled a distance of 176 *cm*, and the other travelled 80 *mm* more than the first. What is the length that both ants travelled in total?

A) 36 *dm* **B**) 260 *cm* **C**) 402 *mm*

Problem 5. The product of 4 natural numbers is 72. The sum of these numbers is 15 and neither of them is 2. Which is the greatest among these numbers?

A) 6 **B**) 8 **C**) 9

Problem 6. We have 5 identical chocolate bars, each consisting of 28 pieces. We have to divide them equally between 7 children. What is the minimum number of times we need to break each chocolate bar in order to do this?

A) 6 **B**) 7 **C**) 8

Problem 7. The sum of the three-digit numbers $\overline{32A}$, $\overline{5B6}$ and $\overline{C11}$ is 1010. (A, B and C represent missing numbers). In this case, what is the three-digit number $\overline{ABC}$?

A) 382 **B**) 371 **C**) 473

Problem 8. A book has been numbered as follows: the first pair of pages has been numbered as 1 and 2; the second pair as 3 and 4, and so on, until the last pair of pages, which has been numbered as 127 and 128. If I open the book at a random place, what is a possible product of the numbers of the two pages that I've opened the book at?

A) 90 **B**) 72 **C**) 56

Problem 9. In the same room we have 2 grandmothers, 4 mothers, 4 daughters, 2 granddaughters. What is the smallest number of people that there could be in the room?

A) 4 **B**) 6 **C**) 8

Problem 10. How many digits are used to write down the first 100 odd numbers?

A) 250 **B**) 245 **C**) 200

Problem 11. Four children met together: Adam, Bobby, Charley and Daniel. Adam shook hands with 3 of these children, Bobby shook hands with 2, and Charley shook hands with 1. How many of the children's hands did David shake?

Problem 12. I solve 6 problems a day, and my brother solves three times less. Together we solved 72 problems. How many days did it take us to do this?

Problem 13. The product of a few different one-digit numbers is a number that has a 5 in the ones place. How many even numbers are there among the multipliers?

Problem 14. Between each two neighbouring digits of the number 2016, I placed either 2 addition signs and 1 multiplication sign, or 2 multiplication signs and 1 addition sign.

Example: $2 + 0 + 1 \times 6$ or $2 \times 0 \times 1 + 6$.

How many different numbers will I get after calculating all such expressions?

Problem 15. Annie has a magical necklace. Each bead of the necklace is numbered (1, 2, 3, 4 and so on). If between the beads numbered as 5 and 15 there is the same number of beads, what is the total number of beads on Annie's necklace?

Problem 16. In Rose's garden there are 232 roses which are not in bloom yet and 168 which are blooming. Every day 4 new roses bloom and the ones that are already blooming do not fade. How many days will it take for the blossoming and non-blossoming roses to be an equal number?

Problem 17. A vessel, when full of water, weighs 20 *kg*, and when half full it weighs as much as 3 empty vessels. How many kilograms does this vessel weigh when it is empty?

Problem 18. A square has a side length of 1 *cm*. On each of its sides (on the outside) has been built another square with a side length of 1 *cm*. After that, on each of the sides of the new figure, another square with a side length of 1 *cm* has been built. How many *cm* is the perimeter of the final figure?

Problem 19. What is the maximum possible number of different odd three-digit numbers that we can add and receive a three-digit number as a result?

Problem 20. The expression we are going to use for this problem is

$$6 \div 2 + 4 \times 3 - 1 \times 10.$$

Exchange one of the numbers from the expression with such a number that the initial value of the expression would be increased by 1. How many of the numbers can be changed?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**A**	$(4+7+9) \times 6 = 4 \times 6 + 7 \times 6 + 9 \times 6 = 24 + 42 + 54 \Rightarrow \square = 54$
2	**C**	$1000 - (12 + 23 + 34 + 45 + 55 + 66 + 77 + 88) =$ $1000 - (100 + 100 + 100 + 100) = 1000 - 400 = 600$
3	**C**	One *kg* of dried mushrooms can be derived from 12 *kg* of fresh mushrooms. In order to get 6 *kg* of dried mushrooms we would need $6 \times 12 = 72$ *kg* fresh mushrooms.
4	**A**	One of the ants travelled a distance of 176 *cm*, and the other traveled $176 + 8 = 184$ *cm*. The distance that both ants travelled in total is equal to $176 + 184 = 360$ *cm* = 36 *dm*.
5	**B**	$15 = 8 + 3 + 3 + 1 = 6 + 6 + 2 + 1 = 9 + 2 + 2 + 2$ Therefore the number we are looking for is 8.
6	**A**	The number of all pieces of all five chocolates is $5 \times 28 = 140$. Therefore each child should get $140 \div 7 = 20$ pieces. By breaking one chocolate, we can get 20 pieces and have 8 left. In this way we can give 20 pieces to 5 children, however there would be 2 more children and 5 more parts, each consisting of 8 pieces, left. We can give 2 parts with 8 pieces each to each of the two children, and the fifth part, which consists of 8 pieces, we can divide in 2 parts of 4 pieces. The number of times we would need to break the chocolates is 5 + 1 = 6.
7	**B**	$\overline{32A} + \overline{5B6} + \overline{C11} = 1010$ $A + 6 + 1 = \cdots 0 \Rightarrow A = 3$ $2 + B + 1 + 1 = \cdots 1 \Rightarrow B = 7$ $3 + 5 + C + 1 = \cdots 0 \Rightarrow C = 1$ Therefore $\overline{ABC} = 371$.
8	**B**	If I open the book, there will be two pages, both numbered. The smaller number will be even, and the greater will be odd. The numbers will be consecutive. $90 = 9 \times 10$; $72 = 8 \times 9$ and $56 = 7 \times 8$, therefore I have opened the book at the pages numbered 8 and 9.
9	**B**	In order for one of the women to be a grandmother, she would need to have a daughter, and a granddaughter. Therefore if there are two

<table>
<tr><td></td><td></td><td>grandmothers, who are also mothers, they have one daughter each, i.e. 2 daughters, each of whom is also a mother to 1 granddaughter – 2 granddaughters, who are also daughters.
The two granddaughters are also 2 daughters.
There are now 2 daughers left, who are also 2 mothers.
There are now 2 mothers left, who are also 2 grandmothers.</td></tr>
<tr><td>10</td><td>B</td><td>Among the first 100 odd numbers there are 5 one-digit numbers, 45 two-digit numbers and 50 three-digit numbers.
Therefore the number of digits which have been used to write them down is $5 \times 1 + 45 \times 2 + 50 \times 3 = 245$.</td></tr>
<tr><td>11</td><td>2</td><td>
<table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>A</td><td></td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>B</td><td>+</td><td></td><td>–</td><td>+</td></tr>
<tr><td>C</td><td>+</td><td>–</td><td></td><td>–</td></tr>
<tr><td>D</td><td>+</td><td>+</td><td>–</td><td></td></tr>
</table>
If we add the number of hand shakes, the number must be divisible by 2, because each hand shake is counted twice.
In this case the number of hand shakes is $6 + x$.
We can mark the number of David’s handshakes with x. The number x can NOT be greater than 3.
$6 + x$ can be divided by 2 only if x is either 0 or 2.
However, x is not 0, because Adam shook hands with all the children. Therefore $x = 2$. David shook hands with 2 children.</td></tr>
<tr><td>12</td><td>9</td><td>I solve 6 problems a day, and my brother solves 2. Together we solved 8 problems in total. It would take us 9 days to solve 72 problems.</td></tr>
<tr><td>13</td><td>0</td><td>The number 5 is among the multipliers. If there is at least one even number, then the product would be divisible both by 2, and by 5, i.e. by 10, it would have a ones digit of 0. Among the multipliers there are no even numbers.</td></tr>
<tr><td>14</td><td>4</td><td>Here are all the different options:
$2 + 0 + 1 \times 6 = 8$; $2 + 0 \times 1 + 6 = 8$; $2 \times 0 + 1 + 6 = 7$;
$2 \times 0 \times 1 + 6 = 6$; $2 \times 0 + 1 \times 6 = 8$; $2 + 0 \times 1 \times 6 = 2$;</td></tr>
</table>

		The result consists of 4 numbers: 2, 6, 7 and 8.
15	**20**	The beads with numbers from 6 to 14 are situated between the beads with numbers from 5 to 15. The beads are 9 in total. The beads on the opposite side are also 9. If we also note the 2 beads numbered 5 and 15 we will find that the beads on Annie's necklace are $2 \times 9 + 2 = 20$.
16	**8**	The roses which are blooming and the roses which are not yet in bloom are 400 in total. The number of the roses in bloom needs to be increased by 32 roses. This will happen in $32 \div 3 = 8$ days.
17	**4**	The weight of the water in a half-full vessel is equal to two empty vessels. The weight of the water in a full vessel weighs as much as 4 empty vessels. The weight of the vessel plus the water inside it is equal to 5 empty vessels. Therefore one empty vessel would be equal to $20 \div 5 = 4$ kg.
18	**20**	There are 4 squares with 3 sides each and 4 squares with 2 sides each which form the final figure. Therefore we get that $4 \times 3 + 4 \times 2 = 20$ *cm*.
19	**9**	$101 + 103 + 105 + 107 + 109 + 111 + 113 + 115 + 117 = 981$; $101 + 103 + 105 + 107 + 109 + 111 + 113 + 115 + 117 + 119 = 1100$.
20	**2**	In order for the value of the expression to be increased by 1, the following needs to be true: The first number in the expression 3+12-10 needs to be exchanged with 4. Then the initial value would be increased by 1. $\square \div 2 = 4$ would be possible if we exchange 6 for 8. $6 \div \square = 4$ is not possible. The second number in the expression 3+12-10 needs to be exchanged with 13. Then the initial value of the expression would be increased by 1. $\square \times 3 = 13$ is not possible. $4 \times \square = 13$ is also not possible. The third number in the expression 3+12-10 needs to be exchanged with 9. Then the initial value would be increased by 1. $1 \times \square = 9$, if we exchange 10 for 9. $\square \times 10 = 9$ is not possible. We can exchange the following, so that the initial value would be increased by 1: $8 \div 2 + 4 \times 3 - 1 \times 10$ or $6 \div 2 + 4 \times 3 - 1 \times 9$

FINAL 2016: GROUP 3

Problem 1. If $\overline{\square 2} \times \Delta = 128$, then $\square \times \Delta = ?$

A) 10 **B**) 12 **C**) 14

Problem 2. $1000 - (5 + 15 + 25 + 35 + 45 + 55 + 65 + 75 + 85 + 95) = ?$

A) 400 **B**) 500 **C**) 600

Problem 3. If we have 1 *kg* of fresh mushrooms and we dry them, we would get 100 *g* of dried mushrooms. How much fresh mushrooms do we need in order to derive 2 *kg* of dried mushrooms?

A) 10 *kg* fresh mushrooms **B**) 20 *kg* fresh mushrooms **C**) 30 *kg* fresh mushrooms

Problem 4. The segment *AB* is 1 *km* long and has been divided into 1000 equal parts by a number of points. The points have been numbered, with *A* being the first point and *B* being the last point. Point *C* is found at an equal distance between point 101 and point 203. What is the distance (in meters) from point *A* to point *C*?

A) 150 **B**) 151 **C**) 152

Problem 5. Iva arrived at the bus stop and looked at her watch, which showed the time to be 08:01h, which meant that she was 2 minutes late for her bus. What she did not know was that her watch was running 5 minutes ahead. If the bus came 1 minute late, how many minutes did Iva have to wait at the bus stop?

A) 4 **B**) 5 **C**) more than 5

Problem 6. Now many times at least would we need to break 6 chocolates, in order to divide them equally between 4 children? Each chocolate is made up of 28 pieces.

A) 2 **B**) 4 **C**) 7

Problem 7. A book is numbered as follows: the pages on the first sheet are numbered as 1 and 2, the pages on the second sheet are numbered as 3 and 4, and so on, until the last sheet, where the pages have been numbered as 47 and 48.

If I were to rip off 3 consecutive sheets and then add the 6 numbers with which the pages of the sheets have been numbered, which of the following sums would I get?

A) 23 **B**) 35 **C**) 45

Problem 8. If we exchange the identical letters with identical numbers, and the different letters with different numbers, then what would the greatest possible value of the following expression be equal to?

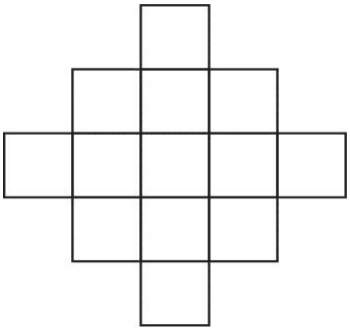
$$N + E + S \times S + E \times B + A - R$$

A) 157 **B**) 156 **C**) 158

Problem 9. Adam wrote down 35 numbers. The first number he wrote is 7, and each next number is twice as big as the preceding one. How many of the numbers he wrote down are greater than 224?

A) 30 **B**) 29 **C**) 28

Problem 10. On the figure below you can see that in the middle there is a square with a side of 1cm. On each of its sides there is another square, each with sides of 1cm. On each of the sides of the newly formed figure, there is one extra square with a side of 1cm. What is the minimum number of squares that must be erased in order for only 15 squares to remain on the figure?



A) 1 **B**) 2 **C**) more than 2

Problem 11. If the first day of the year is a Monday, what would the last day of the same year be?

Problem 12. Alex and Boris each have 3 coins of 1, 2 and 5 cents. Boris used 7 of those coins to add up the smallest possible sum and Alex used 7 of those coins to add up the greatest possible sum. By how much is Boris' sum smaller than that of Alex?

Problem 13. Five people *(A, B, C, D* and *E)* are waiting in a queue. *C* is between *E* and *D*, *A* is next to *E*, and *B* is NOT last. Which one is last?

Problem 14. You are given the following expression: $6 \div 3 + 2 \times 3 - 1 \times 2$. Exchange one of the numbers in the expression with a different number, so that the initial value of the expression would be increased by 1. In how many ways can we do this?

Problem 15. How many times is the number hidden under the first shell smaller than the number hidden under the second shell?

1,  **, 2, 6, 24,**  **, 720, 5 040**

Problem 16. At least how many of the numbers do we need to change in order for the product of the numbers along the diagonals, rows and columns to be the same?

1	4	8
16	4	1
2	4	8

Problem 17. Each of the 10 digits has been used once to write down 5 two-digit numbers with the greatest possible sum. What is the sum?

Problem 18. At a football game, the winner earns 3 points and the loser earns 0 points. If the match is drawn, both teams get 1 point each. After having played 7 games, a team had earned 11 points. What is the possible number of losses that the team had?

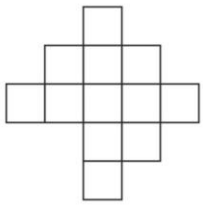
Problem 19. A number is *perfect* when the sum of its divisors (except the number itself) equals the given number. The number 6 is called perfect because it is equal to the sum of

1 + 2 + 3, where 1, 2 and 3 are all its divisors, except the number 6. The next perfect number is an even number greater than 24 and smaller than 30. What is the number?

Problem 20. Two blue whales weigh 300 *tons*, and three seals weigh 1 *ton*. How many seals weigh as much as 1 whale?

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**B**	$332\times4=1328 \Rightarrow \square\times\Delta=12$
2	**B**	$1000 – (5 + 15 + 25 + 35 + 45 + 55 + 65 + 75 + 85 + 95) =$ $= 1000 – 500 = 500$
3	**B**	2 *kg* = 2000 *grams* = 20 × 100 *grams*, which means that we would need 20 *kg* of mushrooms.
4	**B**	Point *C*, which is found at an equal distance between point 101 and point 203, is point 152. The distance from the first point to the 152nd point is 151 meters.
5	**A**	The watch shows that the time is 8:01h. She is 2 minutes late, which means that she was supposed to arrive at 7:59h, according to her watch. This means that she came 3 minutes early, because the bus (according to her watch) was supposed to arrive at 8:04h. The bus is running 1 minute late. Therefore it would arrive at 8:05h. Iva had to wait for 4 minutes.
6	**A**	Each child will receive 1 chocolate plus extra 14 pieces of the remaining chocolates. In this case we would only need to break the chocolates twice.
7	**C**	Everything follows from: $\underbrace{1+2+3+4+5+6=}_{6\ addends}$ 21; $\underbrace{3+4+5+6+7+8=}_{6\ addends}$ 33; $\underbrace{5+6+7+8+9+10=}_{6\ addends}$ 45.
8	**B**	$6+8+9\times9+8\times7+5-0=156$

9	**B**	The numbers are 7, 14, 28, 56, 112, 224, 448,... 5 of them are smaller than 224, 1 is equal to 224, 35 – 6 = 29 are greater than 224.
10	**A**	The 1×1 squares are 13, the 2×2 squares are 4, and there is one 3×3 square. There are 18 squares in total. If we were to remove one 1×1 square, the 1×1 squares would be 12, the 2×2 squares would be 3, and the 3×3 squares would be 0, therefore the total number of squares would be 15. This can be done in 4 ways, by removing 1 square from one of the 4 angles of the the 3×3 square. 
11	**Monday or Tuesday**	The calendar year has 365 days, or 366 days on a leap year. When dividing 364 and 365 by 7, the remainders are 0 and 1. Therefore the last day of the year would either be a Monday or a Tuesday.
12	**8**	The smallest possible sum is $3 \times 1 + 3 \times 2 + 1 \times 5 = 14$, and the greatest possible sum is $3 \times 5 + 3 \times 2 + 1 \times 1 = 22$. The difference is 22 – 14 = 8.
13	***A* or *D***	First we need to arrange 4 people as follows: *DCEA* or *AECD* In this case *B* may be situated as follows: *BDCEA* *DCEAB* *AECDB* *BAECD* However *B* is not last, therefore *A* or *D* can be last.

<table>
<tr><td>14</td><td>3</td><td>$9 \div 3 + 2 \times 3 - 1 \times 2$;
$6 \div 2 + 2 \times 3 - 1 \times 2$;
$6 \div 3 + 2 \times 3 - 1 \times 1$
We can do that in 3 ways.</td></tr>
<tr><td>15</td><td>120</td><td>The rule is as follows: multiply the first number by 1, and get the second; multiply the second number by 2, and get the third, etc.
1; $1 \times 1 = 1$; $1 \times 2 = 2$; $2 \times 3 = 6$; $6 \times 4 = 24$, $24 \times 5 = 120$; $120 \times 6 = 720$; ...
The number hidden under the first shell is 1, and the number hidden under the second shell is 120. $120 \div 1 = 120$.</td></tr>
<tr><td>16</td><td>1</td><td>If we exchange the number in the first row, first column, with a 2, we would get a magical square.<table><tr><td>2</td><td>4</td><td>8</td></tr><tr><td>16</td><td>4</td><td>1</td></tr><tr><td>2</td><td>4</td><td>8</td></tr></table></td></tr>
<tr><td>17</td><td>360</td><td>90+81+72+63+54=360.</td></tr>
<tr><td>18</td><td>0 or 2</td><td>If a team were to win all 7 games, they would earn 21 points. This team however earned 11 points, which means that they lost 10 points from the maximum score.
This can be done through:
5 drawn games and 0 losses,
2 drawn games and 2 losses;
Answer: 0 or 2 losses.</td></tr>
<tr><td>19</td><td>28</td><td>The number is 28. 28 = 1 + 2 + 4 + 7 + 14.</td></tr>
<tr><td>20</td><td>450</td><td>2 blue whales weigh 300 tons, which is equal to 900 walruses. Therefore 1 blue whale weighs as much as 450 walruses.</td></tr>
</table>

TEAM COMPETITION – NESSEBAR, BULGARIA

MATHEMATICAL RELAY RACE

The answers to each problem are hidden behind the symbols **@, #, &, §** and ***** and are used in solving the following problem. Each team, consisting of three students of the same age group, must solve the problems in 45 minutes and then fill a common answer sheet.

GROUP 3

Problem 1. If $\underbrace{4+4+4+\cdots+4}_{@\text{ addends}} = 6\times 6+4$, find **@**.

Problem 2. Our rabbit now has less than **@** male and female bunnies. Each male bunny has as many sisters as brothers, and each female bunny has half as many sisters as brothers. If the number of bunnies our rabbit has is **#**, then find **#**.

Problem 3. Find the smallest even three-digit number **&**, if it is known that $\&-5$ is divisible by **#**.

Problem 4. The number **#**+2 is presented as the product of 4 consecutive odd numbers with a sum of **§**. Find **§**.

Problem 5. There are **§** stacks of hay in a meadow. Four of the stacks have been gathered into one stack and the remaining have been grouped in 3. There are ***** stacks of hay left in the meadow. Find *****.

ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	**@ = 10**	$\underbrace{4+4+4+\cdots+4}_{@\ \text{събираеми}} = 40 \Rightarrow @ = 10$ $\underbrace{4+4+4+\cdots+4}_{@\ \text{addends}} = 40 \Rightarrow \mathbf{@} = 10$
2	**# =7**	The condition says that each male bunny has as many sisters, as brothers. Therefore the number of baby bunnies our rabbit has is 3, 5, 7 or 9. We can check each possible answer: If the correct answer is 3, then each male bunny would have 1 brother and 1 sister. However the only female bunny in this case would not have "half as many sisters as brothers". If the correct answer is 5, then each male bunny would have 2 brothers and 2 sisters. However in this case each female bunny would have 1 sister and 3 brothers, therefore it would not have not have "half as many sisters as brothers". If the correct answer is 7, then each male bunny would have 3 brothers and 3 sisters. In this case each female bunny would have 2 sisters and 4 brothers, which means that the condition is satisfied – each female bunny would have "half as many sisters as brothers". If the correct number is 9, then each male bunny would have 4 brothers and 4 sisters. In this case each female bunny would have 3 sisters and 5 brothers, therefore it would not have "half as many sisters as brothers".
3	**& =103**	If we carry out a check, we will find that among the differences 100−5, 101−5, 102−5, 103−5, 104−5, 105−5,...., the first one that is divisible by #, i.e. by 7, is 103−5. The number we are looking for is 103. **&** = 103.
4	**§ =16**	The number $103 + 2 = 105 = 1 \times 3 \times 5 \times 7$. The sum of these multipliers is 16.
5	*** = 5**	There are 16 stacks of hay. They are stacked as 4 + 3 + 3 + 3 + 3. We can now calculate that the stacks left in the meadow are 1+1+1+1+1 = 5.