**MATHEMATICS WITHOUT BORDERS**

**2015-2016**

## AUTUMN 2015: GROUP 2

**Problem 1.** What is the missing number?

$$10 + ? = 12 + 9$$

**A)** 10 **B)** 11 **C)** 21

**Problem 2.** The sum of 10 + 8 equals:

**A)** the sum of 6 and 11 **B)** the difference of 14 and 4 **C)** the sum of 9 and 9

**Problem 3.** In a sum of two numbers, one addend is greater than 2 by 2, while the other addend is smaller than 1 by 2. The sum is:

**A)** 2 **B)** 4 **C)** 5

**Problem 4.** What is the largest two-digit number with 0 as a units digit?

**A)** 10 **B)** 90 **C)** 100

**Problem 5.** How many of the following expressions are correct?

$$11\text{-}2 > 13$$

$$18+3 > 20$$

$$12\text{-}5 = 3+4$$

**A)** 1 **B)** 2 **C)** 3

**Problem 6.** How many are all the possible digits that can be placed instead of @, so that $36 < 3@$ would be true?

**A)** 9 **B)** 6 **C)** 3

**Problem 7.** What is the largest sum of 2 different single-digit numbers?

**A)** 19 **B)** 18 **C)** 17

**Problem 8.** I thought of a number. I added it to 2 and got 10. The number I thought of is:

**A)** 12 **B)** 8 **C)** 10

**Problem 9.** One of the addends is the smallest two-digit number, and is larger by 1 than the other addend. What is the sum of the two addends?

**A)** 11 **B)** 19 **C)** 21

**Problem 10.** How many are the two-digit numbers that do NOT have 9 as a ones digit?

**A)** 9 **B)** 81 **C)** 90

**Problem 11.** Peter solved 3 problems, Iva solved 2 problems less than Peter; Mary solved one problem more than Iva. How many problems did Mary solve?

**Problem 12.** There is a basket in a dark room. In the basket there are 2 yellow and 3 red apples. What is the smallest possible number of apples we would need to take out, without looking at their colour, in order to ensure that we have taken out 2 red apples?

**Problem 13.** How many single-digits numbers is the magic square made of?

| | | |
|---|---|---|
| 6 | 8 | 1 |
| | 5 | |
| | 2 | |

**Problem 14**. How many sheets of paper are there between the third and the seventh pages of a book?

**Problem 15.** Find the sum of all two-digit numbers whose sum of digits is 3?

**Problem 16.** How many numbers have been omitted in the sequence 1, 11, 21, 31, ..., 81, 91?

**Problem 17.** Joel has a few bunnies. Each one of them has 2 ears and 4 legs. If their ears are 10 in total, how many legs do they have in total?

**Problem 18.** If the minuend is 9 and the subtrahend is 9, we get a difference of?

**Problem 19.** How many units are there in the number equal to

$$9 - 1 + 8 - 2 + 7 - 3 + 6 - 4 + 5 - 5?$$

**Problem 20.** How many sticks with a length of 4 *cm* can we cut off from a stick with a length of 17 *cm*?

## ANSWERS AND SHORT SOLUTIONS

| Problem | Answer | Solution |
|---|---|---|
| **1** | **B** | $10+?=12+9 \Rightarrow 10+?=21 \Rightarrow ?=11$ |
| **2** | **C** | 10 + 8 = 18, 18 = 9 + 9 |
| **3** | **C** | 2 + 2 = 4; 2 – 1 = 1<br>4 + 1 = 5 |
| **4** | **B** | 90 |
| **5** | **B** | 9>13; 21>20; 7=7 |
| **6** | **C** | 36<37; 36<38; 36<39 |
| **7** | **C** | 9 + 8 = 17 |
| **8** | **B** | ? + 2 = 10 ⇒ ? = 8 |
| **9** | **B** | 10; 10 – 1 = 9; 10 + 9 = 19 |
| **10** | **B** | $\underbrace{10,11,\ldots,18}_{9},\ \underbrace{20,21,\ldots,28}_{9},\ \ldots,\ \underbrace{90,91,\ldots,98}_{9}$<br>9+9+9+9+9+9+9+9+9=81 |
| **11** | **2** | Iva solves 1 problem, Maria solved 1 + 1 = 2 problems. |
| **12** | **4** | If we were to take both yellow apples, the next 2 would be red. Therefore if we take 4 apples, there will always be 2 red apples among them. |
| **13** | **8** | 6 8 1<br>0 5 10<br>9 2 4 |
| **14** | **1** | This is the list of paper with page numbers 5 and 6. |
| **15** | **63** | The numbers are 12, 21 and 30. Their sum is 63. |
| **16** | **4** | The numbers 41, 51, 61 and 71 have been skipped. |
| **17** | **20** | There are 10 ears. Therefore the bunnies are 5. Each bunny has 4 legs.<br>4 + 4 + 4 + 4 + 4 = 20. |
| **18** | **0** | 9 – 9 = 0 |
| **19** | **20** | $9-1+8-2+7-3+6-4+5-5=20$ |
| **20** | **4** | 4 + 4 + 4 + 4 = 16 |

**WINTER 2016: GROUP 2**

**Problem 1.** What is the missing number? $(50 + 7) - ? = 56$

**A)** 1 **B)** 2 **C)** 3

**Problem 2.** The sum of $24 + 57 + 9$ is:

**A)** 90 **B)** 80 **C)** 70

**Problem 3.** In a sum of two numbers, one of the addends is greater than 20 by 20, and the other addend is smaller than 20 by 10. The sum of the two numbers is:

**A)** 50 **B)** 40 **C)** 30

**Problem 4.** How many of the following expressions are correct?

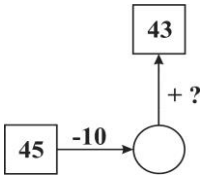
$$40 - 2 = 42$$

$$50 - 18 = 32$$

$$38 + 2 + 10 = 50$$

**A)** 1 **B)** 2 **C)** 3

**Problem 5.** What is the missing number „?”?




**A)** 8 **B)** 18 **C)** 35

**Problem 6.** How many digits can we place instead of @, so that $44 - 4 > 4@$ would <u>not be true</u>?

**A)** 10 **B)** 9 **C)** 1

**Problem 7.** What is the greatest sum of 3 different one-digit numbers?

**A)** 23 **B)** 24 **C)** 25

**Problem 8.** There is a basket in a dark room. In the basket there are 6 yellow and 5 red apples. What is the <u>smallest</u> possible number of apples we would need to take out, without looking at their colour, in order to ensure that we have taken out at least 3 red apples?

**A)** 8 **B)** 9 **C)** 10

**Problem 9.** If we add the number equal to 94 – (46 + 38) to the number equal to 94 – 46 +38, what result would we get?

**A)** 86 **B)** 76 **C)** 96

**Problem 10.** A gallery has 96 paintings. 32 of them were sold on the first day, and on the second day the gallery sold 3 paintings more than the previous day. How many paintings are still not sold?

**A)** 61 **B)** 39 **C)** 29

**Problem 11.** Three friends weigh respectively 24, 30 and 42 kilograms. They want to cross a river by using a boat that can carry a maximum of 70 kg. At least how many times would this boat need to cross the river, so that all three of them would get to the opposite shore

**Problem 12.** How many tens are there in the number equal to

$$92 - 72 + 83 - 63 + 74 - 34 + 65 - 45 + 56 - 56?$$

**Problem 13.** What is the greatest number in the magic square?

| | | |
|---|---|---|
| 6 | 8 | 1 |
| | | |
| | 2 | |

**Problem 14**. In how many squares can you find the letter A?

| | | |
|---|---|---|
| | | |
| A | | |
| | | |

**Problem 15.** Place the digits 1, 2, 3 and 4 in the squares in a way that would result in the greatest sum.

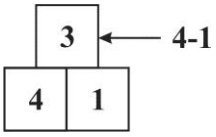
□ + □ + □□. What is the sum?

**Problem 16.** Boko and Tsoko went fishing with their sons. All of them caught an equal number of fish. How much fish did each of them catch, if they caught 9 fish in total?

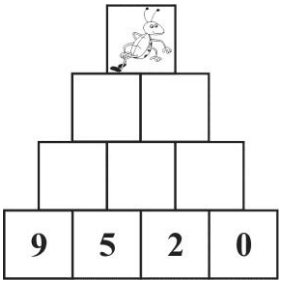
**Problem 17.** The minuend is greater than the subtrahend by 2. What is the difference?

**Problem 18.** How many are the three digit numbers different from 102, that can be derived from the number 102 by randomly moving the digits of the number around?

**Problem 19.** If we follow the rule:




then which number do we need to place in the square with the ant in it?




**Problem 20.** How many are the numbers smaller than 101?

## ANSWERS AND SHORT SOLUTIONS

| Problem | Answer | Solution |
|---|---|---|
| **1** | **A** | 57 – ? = 56; ? = 1 |
| **2** | **A** | 90 |
| **3** | **A** | One of the addends is 20 + 20 = 40, and the other is 20 – 10 = 10. The sum is 50. |
| **4** | **B** | 40 – 2 = 38, i.e. the first expression is not correct. The next two expressions are correct. |
| **5** | **A** | The missing number in the circle is 35. Then we must add 8 to the number 35, in order to get 43.<br>The number we are looking for is 8. |
| **6** | **A** | We need to find out the following: for how many digits @ is it NOT true that: 40 > 4@?<br>For all ten digits: 0, 1, ..., 9. |
| **7** | **B** | 9 + 8 + 7 = 24 |
| **8** | **B** | In the worst case scenario, we would take out all of the yellow apples first. Then after 3 more attempts, we would have taken out 3 red apples, i.e. 9 in total. |
| **9** | **C** | The first addend is 10, and the second is 86. The sum is 96. |
| **10** | **C** | The paintings sold on the second day were 35. The paintings sold on the first and second day together are 67.<br>The paintings that remain unsold are 96 – 67 = 29. |
| **11** | **3** | Let *C* denotes the heaviest of the three friends, *A* - the lightest one, and *B* - the third one.<br>It would be impossible for all three of them to cross the river in one go, because 24 + 30 + 42 = 96 > 70.<br>Therefore the boat would have to return at least once, and the smallest possible number of river crossings would be 3.<br>Following is an example of a way in which all three friends can cross the river to the opposite shore:<br>C stays on one of the shores, while A and B cross over to the opposite shore. |

| | | |
|---|---|---|
| | | A crosses back to the initial shore.<br>A and C now cross to the opposite shore together. |
| **12** | **10** | $(92 - 72) + (83 - 63) + (74 - 34) + (65 - 45) + (56 - 56) =$<br>= 20 + 20 + 40 + 20 + 0 = 100. In the number 100 there are 10 tens. |
| **13** | **10** | The magical sum is 15.<br>The numbers in the second row are 0, 5 and 10, and in the third row are 9, 2 and 4.<br>The greatest number is 10. |
| **14** | **4** | The letter A is in one square 1×1, in two squares 2×2 and in one square 3×3. |
| **15** | **46** | 1 + 2 + 43 = 46 |
| **16** | **3 or 1** | If we assume that the problem speaks of four people – two fathers and two sons, then the result would be impossible, because 9 is not divisible by 4. Therefore the problem must speak of three people: a grandfather, his son, and his grandson, or of 9 people: two fathers and seven sons. |
| **17** | **2** | □ + 2 – □ = 2 |
| **18** | **3** | The numbers are 102, 120, 201 and 210.<br>One of them has been written down already. |
| **19** | **0** | The numbers are as follows:<br>At the bottom: 9, 5, 2, 0<br>Above: 4, 3, 2<br>Above: 1, 1<br>And the number at the top is 0. |
| **20** | **101** | The numbers smaller than 101 are the numbers from 0 to 100. 101 in total. |

**SPRING 2016: GROUP 2**

**Problem 1** If $100 - (29 + 37) = \square - 65$, then $\square$ =

**A)** 100 **B)** 99 **C)** 98

**Problem 2.** Which of the following lengths is the shortest?

**A)** 3 *mm* **B)** 2 *cm* **C)** 1 *dm*

**Problem 3.** If $2 + 3 \times 5 = 5 \times 5 - \square$, then $\square$ =

**A)** 0 **B)** 9 **C)** 8

**Problem 4.** I chose a random number. I switched the numbers of ones and tens. I added 19 to the resulting number and got 24. What is the number I had originally chosen?

**A)** 15 **B)** 50 **C)** 51

**Problem 5.** Alia and Daniel had 24 sweets at first. Then Alia bought 2 more sweets and she now has 12 sweets more than Daniel. How many sweets does she have at the moment?

**A)** 18 **B)** 19 **C)** 20

**Problem 6.** The even numbers from 3 to $\square$, inclusive, is 20. What is the greatest possible value of $\square$?

**A)** 41 **B)** 42 **C)** 43

**Problem 7.** Which of the following numbers is the smallest?

**A)** $3 + 2 \times 2$ **B)** $13 - 3 \times 1$ C) $(3 + 2) \times 2$

**Problem 8.** The number of sparrows on each tree is equal to the total number of trees. The total number of sparrows is 16. How many trees are there?

**A)** 3 **B)** 4 **C)** 5

**Problem 9.** Two two-digit numbers have been written using 4 different digits. Which of the following sums is possible?

**A)** 22 **B)** 26 **C)** 33

**Problem 10.** I bought 9 stamps, worth 6 cents each, and I payed using 6 coins of 10 cents. In how many different ways can I get my change?

**A)** in 3 different ways **B)** in 4 different ways **C)** in 5 different ways

**Problem 11.** The numbers 1, 2, 3, 4 and 6 are written down on two pieces of paper. The product of the numbers from one of the pieces is equal to the product of the numbers from the other piece. How many numbers are there on the piece of paper that has the number 1?

**Problem 12.** There are 2 grandmothers, 4 mothers, 4 daughters and 2 granddaughters in a room. What's the smallest possible number of people in that room?

**Problem 13.** There are 22 students in a class. Twelve of the students have the highest grade in less than four subjects, and 12 have the highest grade in more than two subjects. How many students have the highest grade in exactly three subjects?

**Problem 14.** In Rose's garden there are 88 roses which are not in bloom yet and 8 which are blooming. Every day 4 new roses bloom and the ones that are already blooming do not fade. How many days will it take for the blossoming and non-blossoming roses to be an equal number?

**Problem 15.** Replace the smileys with two of the cards in order to get the greatest possible product. ☺ × ☺


What is the greatest possible product?

**Problem 16**. The square is *'magical'*. Calculate the number A.

| | | |
|---|---|---|
| | 21 | 18 |
| 27 | 15 | A |
| | | 24 |

**Problem 17.** If $\underbrace{2+2+\cdots+2}_{10} = \underbrace{4+4+\cdots+4}_{\square}$ , then $\square$ = ....

**Problem 18.** The product of five numbers is 5. What is their sum?

**Problem 19.** A container full of water weighs 20 *kg* and when half full it weighs as much as 3 empty containers. How many kilograms does this container weigh when it is empty?

**Problem 20.** Four children met together: Adam, Bobby, Charley and Daniel. Adam shook hands with 3 of these children, Bobby shook hands with 2, and Charley shook hands with 1. How many of the children's hands did David shake?

## ANSWERS AND SHORT SOLUTIONS

| Problem | Answer | Solution |
|---|---|---|
| **1** | **B** | 100 –(29 + 37) = ☐ – 65, then 100 – 66 = ☐ - 65 ⇒34 = ☐ - 65 ⇒ ☐ = 99 |
| **2** | **A** | **A)** 3 *mm* **B)** 2 *cm* = 20 *mm* **C)** 1 *dm* = 10 *cm* = 100 *mm* |
| **3** | **C** | If $2 + 3 \times 5 = 5 \times 5 - \square$, then $2 + 15 = 25 - \square \Rightarrow$ $17 = 25 - \square \Rightarrow \square = 8$ |
| **4** | **B** | The number with exchanged digits of the ones and tens is 24 – 19 = 5. Therefore the originally chosen number is 50. From 50 we can get 05 = 5 and 5 + 19 = 24. |
| **5** | **B** | Before buying the extra sweets, Alia had 10 sweets more than Daniel. 24 – 10 = 14 and 14 ÷ 2 = 7, therefore before buying the extra sweets Alia had 17 sweets and Daniel had 7. At the moment Alia has 19 sweets. |
| **6** | **C** | The 20 even numbers from 3 onwards are 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42. The even numbers from 3 to 42, inclusive, are 20. The even numbers from 3 to 43, inclusive, are 20. The even numbers from 3 to 44, inclusive, are 21. |
| **7** | **A** | $3 + 2 \times 2 = 3 + 4 = 7$; $13 – 3 \times 1 = 13 – 3 = 10$; $(3 + 2) \times 2 = 10$ |
| **8** | **B** | If the trees are 3, the sparrows would be $3 \times 3 = 9$; If the trees are 4, the sparrows would be $4 \times 4 = 16$; If the trees are 5, the sparrows would be $5 \times 5 = 25$. |
| **9** | **C** | The sum of two two-digit numbers with different digits, i.e. 1☐ +2Δ, is greater than 30. The only possible option is 33. 33 = 13 + 20. |

| | | |
|---|---|---|
| **10** | **C** | The change would be equal to $6 \times 10 - 9 \times 6 = 6$ cents. It can be given in 5 different ways:<br>$5 + 1 = 2 + 2 + 2 = 2 + 2 + 1 + 1 = 2 + 1 + 1 + 1 + 1 = 1 + 1 + 1 + 1 + 1 + 1$. |
| **11** | **3** | The product of the numbers is equal to $1 \times 2 \times 3 \times 4 \times 6 = 144$. Therefore we would need to write numbers that have a product of 12 on the pieces of paper.<br>The numbers can be written down as follows: 1, 3 and 4 on the first piece of paper, 2 and 6 on the second piece of paper, or 3 and 4 on the first piece of paper, 1, 2 and 6 on the second piece of paper. The pieces of paper that has the number 1 on it has 3 numbers written on it. |
| **12** | **6** | In order for one of the women to be a grandmother, she would need to have a daughter, and a granddaughter. Therefore if there are two grandmothers, who are also mothers, they have one daughter each, i.e. 2 daughters, each of whom is also a mother to 1 granddaughter – 2 granddaughters, who are also daughters.<br>The two granddaughters are also 2 daughters.<br>There are now 2 daughters left, who are also 2 mothers.<br>There are now 2 mothers left, who are also 2 grandmothers. |
| **13** | **2** | The total number of students in the class plus the number of students who have the highest grade in 3 subjects equals $12 + 12 = 24$. If we calculate $24 - 22$ we would get the number of students who have the highest grade in 3 subjects, i.e. 2. |
| **14** | **10** | The roses in blossom and those not yet in blossom are 96 in total. The number of roses in blossom must increase by $96 \div 2 - 8 = 40$ roses. That can happen in $40 \div 4 = 10$ days. |
| **15** | **63** | The possible products are $2 \times 6$; $2 \times 7$; $6 \times 7$; $2 \times 9$; $7 \times 9$. The greatest among them is 63.<br>(We get the number 9 when we turn the card that has 6 written on it.) |

<table>
<tr><td>16</td><td>3</td><td>We can find the answer by comparing the sums of the numbers from the first column (B, 27, C) to the diagonal (B, 15, 24).<br>They are equal, B + 27 + C = B + 15 + 24, therefore 27 + C = 39.<br>We get that C = 12, therefore the ‘magical’ sum is 45 (12 + 15 + 18).<br>27 + 15 + A = 45, therefore A = 3.</td></tr>
<tr><td>17</td><td>5</td><td>If $\underbrace{2+2+\cdots+2}_{10} = \underbrace{4+4+\cdots+4}_{\square}$,<br>then 20 = 4 × □ ⇒□ = 5.</td></tr>
<tr><td>18</td><td>9</td><td>5 = 5 ×1 × 1 ×1 × 1,<br>therefore the sum we are looking for is 5 + 1 + 1 + 1 + 1 = 9.</td></tr>
<tr><td>19</td><td>4</td><td>The weight of the water in a half-full vessel is equal to two empty vessels. The weight of the water in a full vessel weighs as much as 4 empty vessels. The weight of the vessel plus the water inside it is equal to 5 empty vessels.<br>Therefore one empty vessel would be equal to 20÷5 = 4 kg.</td></tr>
<tr><td>20</td><td>2</td><td>
<table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>A</td><td></td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>B</td><td>+</td><td></td><td>–</td><td>+</td></tr>
<tr><td>C</td><td>+</td><td>–</td><td></td><td>–</td></tr>
<tr><td>D</td><td>+</td><td>+</td><td>–</td><td></td></tr>
</table>
If we add the number of hand shakes, the number must be divisible by 2, because each hand shake is counted twice.<br>In this case the number of hand shakes is 6 + x.<br>We can mark the number of David’s handshakes with x. The number x can NOT be greater than 3.<br>6 + x can be divided by 2 only if x is either 0 or 2.<br>However, x is not 0, because Adam shook hands with all the children.<br>Therefore x = 2. David shook hands with 2 children.</td></tr>
</table>

**FINAL 2016: GROUP 2**

**Problem 1.** The product of all even one-digit numbers that are divisible by 3, is:

**A)** 0 **B)** 6 **C)** 18

**Problem 2.** What number should be placed instead of ☐ so that the following equation would be true?

$$1 \times 3 + 3 \times 5 + 5 \times 7 + 7 \times 9 = 2 \times 2 + 4 \times 4 + 5 \times 5 + 6 \times 6 + \square?$$

**A)** 35 **B)** 36 **C)** 37

**Problem 3.** When Adam was counting the numbers from 1 to 50, he got distracted and he forgot to count the numbers that are divisible by 2 or by 3. How many numbers, smaller than 31, did he forget to count?

**A)** 20 **B)** 10 **C)** 5

**Problem 4.** There are 20 odd numbers from 2 to ☐, inclusive. What is the greatest possible value of ☐?

**A)** 41 **B)** 42 **C)** 43

**Problem 5.** Adam, Bobby, Charles and Daniel won the top four places at a competition. Adam was ranked higher than Bobby, Charles was ranked lower than Daniel, and Bobby was ranked higher than Daniel. Who came third?

**A)** Adam **B)** Bobby **C)** Daniel

**Problem 6.** A container full of water weighs 21 *kg* and when half full it weighs as much as 4 empty containers. How many kg of water are there in the container when it is full?

**A)** 3 **B)** 16 **C)** 18

**Problem 7.** When I grow 8 years older than I am now, I will be twice as old as my brother who was born 2 years ago. How old am I at the moment?

**A)** 10 **B)** 12 **C)** 20

**Problem 8.** How many numbers can we place in the empty square, so that the following equation would be true?

$$\square \times 5 < 25?$$

**A)** 4 **B)** 5 **C)** more than 5

**Problem 9.** By how much is the number hidden under the first shell smaller than the number hidden under the second shell?

**4, 7, 13,**  **, 34,**  **, 67**

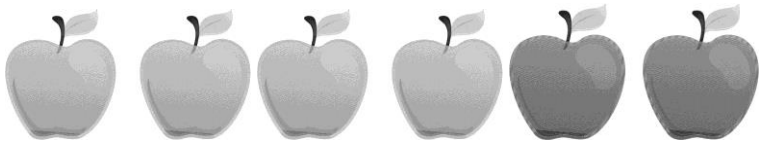
**A)** 7 **B)** 37 **C)** 27

**Problem 10.** We are given the numbers 1, 2, 3 and 4. If we erase two of them, then the product of the remaining numbers can be presented as the product of two equal multipliers. Which numbers should we erase to do that?

**A)** 1 and 2 **B)** 2 and 3 **C)** 2 and 4

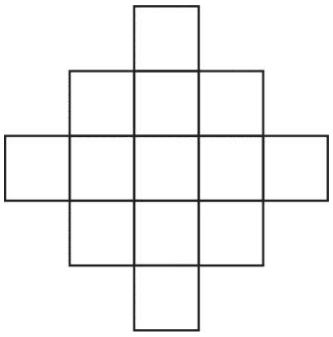
**Problem 11.** A few football teams are participating in a tournament. After a game has been played, only the winner moves forward into the tournament. If the teams are 16, what is the minimum number of games that must be played in order for one of the teams to become a champion?

**Problem 12.** There is a basket of apples in a dark room. There are 4 yellow and 2 red apples inside it. What is the minimum number of apples you would need to take out (without looking) in order to be sure that you have taken out 2 yellow and 1 red apples?



**Problem 13.** The sum of 11 one-digit numbers is 98. What is the smallest among these numbers?

**Problem 14.** On the figure below you can see that in the middle there is a square with a side of 1cm. On each of its sides there is another square, each with sides of 1cm. On each of the sides of the newly formed figure, there is one extra square with a side of 1cm. How many squares are there in total on the figure?



**Problem 15.** Here is what a few children said about the number 63:

Adam: “This is a number made up of odd numbers!”

Bryan: “This number is a product of the numbers 7 and 9!”

Steve: “This number has 63 units!”

How many of the statements above are true?

**Problem 16.** I bought 9 sweets, each of which costs 7 cents, and I paid using 7 coins of 10 cents. In how many different ways can the shopkeeper give me my change?

**Problem 17.** There are 26 students in a class of second-graders. 15 of them have less than four balloons, and 17 have more than two balloons. How many of the students have more than three balloons?

**Problem 18.** What is the smallest possible sum of the numbers that we would need to place in the 6 empty squares, so that the sum of the numbers in order of rows, diagonals, and columns would be the same?

| | | |
|---|---|---|
| | 2 | |
| | | 2 |
| 2 | | |

**Problem 19.** The digits used to write down the even two-digit numbers are more than the digits used to write down the odd one-digit numbers. By how many?

**Problem 20.** John arranged 100 books one next to the other. The book on insects was 29th from left to right, and the book on birds was 82nd from right to left. What is the number (from left to right) of the book that is in the middle of the book on insects and the book on birds?

## ANSWERS AND SHORT SOLUTIONS

| Problem | Answer | Solution |
|---|---|---|
| **1** | **A** | The even one-digit numbers divisible by 3 are 0 and 6.<br>Their product is 0. |
| **2** | **A** | We can write down the equality as follows:<br>3 + 15 + 35 + 63 = 4 + 16 + 25 + 36 + ☐⇒ ☐ = 35 |
| **3** | **A** | He forgot to count all even numbers, of which there are 15, as well as all odd numbers divisible by 3, which are 3, 9, 15, 21 and 27.<br>He forgot to count 20 numbers in total. |
| **4** | **B** | The 20 odd numbers from 2 onwards are 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39,41.<br>The odd numbers from 2 to 41, inclusive, are 20.<br>The even numbers from 2 to 42, inclusive, are 20. |
| **5** | **C** | The four boys are ranked as follows: *AB* and *DC*, therefore *ABDC*. |
| **6** | **C** | If the half-full container weighs as much as 4 empty containers, then the weight of the water in a half-full container is equal to the weigh of 3 empty containers. The weight of the water in a full container is equal to the weight of 6 empty containers. I.e. when full of water, the container weighs as much as 7 empty containers. Therefore one empty container weighs $21 \div 7 = 3kg$.<br>The water in a full container weighs 21 – 3 = 18 kg. |
| **7** | **B** | We can find the correct answer by checking each possible answer. If I am 10 years old now, then my brother is 2 years old. In 8 years I will be 18 and my brother will be 10. The number 18 is not twice as big as 10. If I am 12 now, my brother is 2. In 8 years I will be 20 and he will be 10. This is the correct answer. |
| **8** | B | All numbers from 0 to 4 (5 numbers). |
| **9** | C | **4, 7, 13,**  **, 34,**  **, 67**<br>7 = 4 + 1 × 3;<br>13 = 7 + 2 × 3;<br>If the number hidden under the first shell is 13 + 3 × 3 = 22, then the next number is 22 + 4 × 3 = 34. |

<table>
<tr><td></td><td></td><td>The next number is $34 + 5 \times 3 = 49$,<br>The next number is $49 + 6 \times 3 = 67$.<br>The difference we are looking for is $49 – 22 = 27$.</td></tr>
<tr><td>10</td><td>B</td><td>If we were to erase the numbers 2 and 3, we would get a product of 4, which we can present as $2\times 2$.</td></tr>
<tr><td>11</td><td>15</td><td>First we split the 16 teams into 8 couples.<br>They play 8 games, therefore there are 8 winners.<br>8 teams carry on to the second round.<br>8 teams play 4 games in the second round.<br>4 teams carry on to the third round, to play 2 games.<br>Final: 2 teams play 1 game.<br>The games played in total are $8 + 4 + 2 + 1 = 15$.</td></tr>
<tr><td>12</td><td>5</td><td>In the worst case scenario we would take out all 4 yellow apples first, and the 5th apple would be red.</td></tr>
<tr><td>13</td><td>8</td><td>The sum of 11 one-digit numbers can be 99 at most. In this case it is 98.<br>Therefore, one of them is 8.</td></tr>
<tr><td>14</td><td>18</td><td>There are 14 squares with a side of 1 cm on the figure; 4 squares with a side of 2 cm and 1 square with a side of 3 cm. There are 18 squares in total.</td></tr>
<tr><td>15</td><td>2</td><td>Only Adam’s claim is not true.</td></tr>
<tr><td>16</td><td>6</td><td>The change is 70-63=7 cents.<br>I can get my change in 6 different ways:<br>7 coins of 1 cent;<br>5 coins of 1 cent + 1 coin of 2 cents;<br>3 coins of 1 cent + 2 coins of 2 cents;<br>2 coins of 1 cent + 1 coin of 5 cents;<br>1 coin of 1 cent + 3 coins of 2 cents;<br>1 coin of 2 cents + 1 coin of 5 cents.</td></tr>
<tr><td>17</td><td>11</td><td>15+17-26=6 children have 3 balloons each. 17-6=11 students have more than 3 balloons each.</td></tr>
<tr><td>18</td><td>3</td><td><table><tr><td>1</td><td>2</td><td>0</td></tr><tr><td>0</td><td>1</td><td>2</td></tr><tr><td>2</td><td>0</td><td>1</td></tr></table>The smallest possible sum is 3.</td></tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>We can compare the sums of the numbers from the first row and from the first column. They are equal, therefore they must have two more equal numbers each - x.
<table>
<tr><td></td><td>2</td><td>x</td></tr>
<tr><td>x</td><td></td><td>2</td></tr>
<tr><td>2</td><td></td><td></td></tr>
</table>
We can then compare the second row and the second column:
<table>
<tr><td></td><td>2</td><td>x</td></tr>
<tr><td>x</td><td></td><td>2</td></tr>
<tr><td>2</td><td>x</td><td></td></tr>
</table>
We can then compare the first row with the second column and the first row with the third column and we would get the following:
<table>
<tr><td>y</td><td>2</td><td>x</td></tr>
<tr><td>x</td><td>y</td><td>2</td></tr>
<tr><td>2</td><td>x</td><td>y</td></tr>
</table>
If we then compare the sums along the diagonals, we will get that<br>
$y + y = 2 + x \Rightarrow$ the smallest possible value is $y=1$ and $x=0$.<br>
The sum we are looking for is 3.</td>
</tr>
<tr>
<td>19</td>
<td>85 or 5</td>
<td>There are 45 even two-digit numbers and they have been written down using 90 digits. The odd one-digit numbers are written down using 5 digits. The answer we are looking for is 90 – 5 = 85.<br>
Another answer is also possible:<br>
The even two-digit numbers are written down using 10 digits and the odd one-digit numbers are written down using 5 digits. In this case the answer would be 10 – 5 = 5.</td>
</tr>
<tr>
<td>20</td>
<td>24</td>
<td>The book on birds is 19th from left to right, and the book on insects is 29th. There are 9 books between them. The book in the middle is the 24th book from left to right.</td>
</tr>
</table>

**TEAM COMPETITION – NESSEBAR, BULGARIA**
**MATHEMATICAL RELAY RACE**

The answers to each problem are hidden behind the symbols **@, #, &, §** and ***** and are used in solving the following problem. Each team, consisting of three students of the same age group, must solve the problems in 45 minutes and then fill a common answer sheet.

**GROUP 2**

**Problem 1.** The number of two-digit numbers that can be presented as a product of two consecutive numbers is @. Find @.

**Problem 2.** If the dividend is $\underbrace{2+4+6+\cdots}_{@\text{ consecutive even numbers}}$, and the divisor is 7, what is the quotient #?

**Problem 3.** Little Red Riding Hood needs to cross a river by going through the only bridge, in order to get to her grandmother's village. She can reach the bridge using **&** different roads, and she can use two different roads from the bridge to her grandmother's village. It turns out she can reach her grandmother's village using # different routes. Find **&**.

**Problem 4.** Bugs Bunny loves eating cabbage and carrots. He eats either **&**+1 carrots or 4 cabbages every day. In one week Bugs Bunny ate 30 carrots and **§** cabbages. Find **§**.

**Problem 5.** Four chess players are participating in a chess tournament. The first player has so far played 3 games, and the second and third players, who haven't played each other yet, have so far played **§** games in total. The fourth player has so far played ***** games. Find *****.

## ANSWERS AND SHORT SOLUTIONS

| Problem | Answer | Solution |
|---|---|---|
| **1** | **@ = 7** | $2 \times 3 = 6 < 10$;<br>$3 \times 4 = 12$; $4 \times 5 = 20$; $5 \times 6 = 30$; $6 \times 7 = 42$;<br>$7 \times 8 = 56$; $8 \times 9 = 72$; $9 \times 10 = 90$;<br>$10 \times 11 = 110 > 99$.<br>The number we are looking for is **@ =7.** |
| **2** | **# = 8** | The dividend is $2 + 4 + 6 + 8 + 10 + 12 + 14$.<br>The quotient is $56 \div 7 = 8$. |
| **3** | **& = 4** | **&×2 = #** = 8⇒ **&** = 4. |
| **4** | **§ =4** | Bugs Bunny eats 5 carrots a day. It would take him 6 days to eat 30 carrots. He would only eat cabbage on the seventh day – he would have to eat 3 cabbages.<br>**§**=4. |
| **5** | *** = 3** | We found that **§**=4.<br>Two of the chess players who have not played against each other have played a total of 4 games. From 4=1+3 = 2+2 and the fact that they have not played against each other, it follows that they have each played 2 games – with the first and the fourth players. Therefore the fourth player has played all of his three games.<br>*=3. |