



MATHEMATICS WITHOUT BORDERS  
2015-2016

WINTER 2016: GROUP 1

**Problem 1.** Which of the following symbols “<”, “>” or “=” should we place in the square, so that the equation would be true?

$$1+1+3 \square 6-2$$

A) <

B) >

C) =

**Problem 2.** What is the missing number?

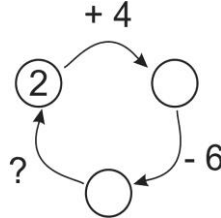
$$\begin{array}{ccc} \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} & \begin{array}{|c|c|} \hline \bullet & \\ \hline \bullet & \\ \hline \end{array} & \begin{array}{|c|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline \end{array} \\ \hline \begin{array}{|c|} \hline 4 \\ \hline \end{array} & + & \begin{array}{|c|} \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline 6 \\ \hline \end{array} \end{array}$$

A) one

B) two

C) three

**Problem 3.** If



? =

A) +7

B) -7

C) +2

**Problem 4.** If

$$\odot + \odot + \square = 10$$

$$\odot + \square = 7$$

$$\square - \odot = ?$$

? =

A) 1

B) 2

C) 3

**Problem 5.** Replace the smileys with two of the cards in order to get the greatest possible sum.

$$\odot + \odot$$



What is the greatest sum?

A) 9

B) 13

C) 16

**Problem 6.** The number of leaves on a few three-leaf clovers can NOT be:



A) 11

B) 12

C) 15

**Problem 7.** How many of the following expressions are correct?

$$1 - 1 = 0$$

$$9 - 1 = 10$$

$$11 + 1 - 10 = 10$$

A) 1

B) 2

C) 3

**Problem 8.** Sonya has 2 fish. Amina has 2 fish more than Sonya. How many fish do Amina and Sonya have in total?

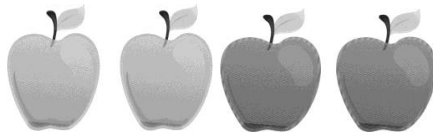


A) 4

B) 6

C) 8

**Problem 9.** There is a basket in a dark room. In the basket there are 2 yellow and 2 red apples. What is the smallest possible number of apples we would need to take out, without looking at their colour, in order to ensure that we have taken out at least 1 red apple?



**Problem 10.** Place the digits 1, 2 and 3 in the squares in a way that would result in the smallest sum. What is the sum?

$$\square + \square \square$$

## ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	B	$1 + 1 + 3 > 6 - 2$
2	B	$4 + 2 = 6$ Two is the missing number.
3	C	$2 + 4 = 6$ ; $6 - 6 = 0$ ; $0 + 2 = 2$
4	A	$\odot + 7 = 10 \Rightarrow \odot = 3$ $3 + \square = 7 \Rightarrow \square = 4$ $\square - \odot = 4 - 3 = 1$
5	C	If we turn the second card upside down, we will get the greatest sum $9 + 7 = 16$
6	A	One three-leaf clover has 3 leaves; two three-leaf clovers have 6 leaves; three clovers have $3+3+3=9$ leaves; four clovers have $3+3+3+3=12$ leaves; five clovers have $3+3+3+3+3=15$ leaves.
7	A	Out of the three expressions only the first is correct.
8	B	Amina has 4 fish, and together with Sonya they have 6 fish in total.
9	3	If the first two apples are yellow, the third would be red.
10	15	$3 + 12 = 15$

**SPRING 2016: GROUP 1**

**Problem 1.**

$$9 + 1 + 2 = 1\square$$

$$\square =$$

A) 1

B) 2

C) 3

**Problem 2.** Which of the following is NOT true?

A)  $9 + 1 + 2 = 12$

B)  $9 + 2 + 2 = 13$

C)  $9 + 2 + 4 = 14$

**Problem 3.** Which of the following are one-digit numbers?

0, 1, 2, 12, 3, 13 and 14

A) 7

B) 4

C) 2

**Problem 4.**

$$9 + 5 = 16 - \square$$

$$\square =$$

A) 1

B) 2

C) 3

**Problem 5.** By how much is 10 greater than 9?

A) 19

B) 1

C) 2

**Problem 6.** The sparrows on each tree are as many as the total number of trees. The total number of sparrows is 4. How many trees are there?

A) 2

B) 3

C) 4

**Problem 7.** I have 17 roses – white, yellow and red. The white and yellow roses together are 10, the yellow and red roses together are 10. How many yellow roses are there?

A) 7

B) 3

C) 1

**Problem 8.**

$$6 + \square = 15$$

$$\square =$$

A) 8

B) 9

C) 10

**Problem 9.** I have 1 apple, Yvette has 1 apple more than me, and Daria has 1 apple less than Yvette. How many apples in total do the three of us have?

A) 4

B) 3

C) 2

**Problem 10.** Nine children are playing hide and seek. One of them is seeking the others and finds 7 of the children. How many of the children remain hidden?

A) 16

B) 2

C) 1

**Problem 11.** I added two different numbers and the sum I got is 2. Which of the two addends is greater?

**Problem 12.** Rather than deducting 1 from a certain number, I added 1 and got 2 as a result. What is the number I should have gotten?

**Problem 13.** Three identical balloons cost 10 cents more than a single balloon of the same kind. How many cents does a single balloon cost?

**Problem 14.** One addend is greater than 1 by 1 and the other addend is less than 2 by 2. What is the sum?

**Problem 15.** I wrote down all numbers from 5 to 16. How many are the digits used more than once?

**Problem 16.** John needs 3 more balloons in order to have 4. Peter has 1 balloon more than John. How many balloons do Peter and John have in total?

**Problem 17.**

$$\square = 1$$

$$\square + \Delta = 4$$

$$\Delta + \text{☺} = 9$$

$$\text{☺} = ?$$

**Problem 18.** There are 6 flowers in a vase, each of which has 3 petals. I picked 4 of the flowers. What is the total number of petals on the remaining flowers?



**Problem 19.** A bunny eats 2 carrots every day. How many days will it take for the bunny to eat 6 carrots?

**Problem 20.**

$$\square \Delta - 3 = 9$$

$$\square + \Delta = ?$$

## ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
<b>1</b>	<b>B</b>	$9 + 1 + 2 = 12 \Rightarrow \square = 2$
<b>2</b>	<b>C</b>	A) true      B) true      C) NOT true $9+2+4=16$
<b>3</b>	<b>B</b>	The one-digit numbers are 0, 1, 2 and 3 $\Rightarrow$ there are four in total.
<b>4</b>	<b>B</b>	$9 + 5 = 16 - \square \Rightarrow 16 - \square = 14 \Rightarrow \square = 2$
<b>5</b>	<b>B</b>	$10 - 9 = 1$
<b>6</b>	<b>A</b>	If there is 1 tree, there would be 1 sparrow; If there are 2 trees then the sparrows would be $2 + 2 = 4$ .
<b>7</b>	<b>B</b>	$\underbrace{\text{White} + \text{yellow}}_{10} + \text{red} = 17 \Rightarrow \text{red} = 7$ $\text{yellow} + \underbrace{\text{red}}_7 = 10 \Rightarrow \text{yellow} = 3 \Rightarrow$ There are 3 yellow roses.
<b>8</b>	<b>B</b>	$6 + \square = 15 \Rightarrow \square = 9$
<b>9</b>	<b>A</b>	I have 1. Yvette has $1 + 1 = 2$ , and Daria has $2 - 1 = 1$ . Together we have $1 + 2 + 1 = 4$ .
<b>10</b>	<b>C</b>	Among the 9 children playing hide and seek, there are 8 hidden children. 7 of them have been found. The children that remain hidden are $8 - 7 = 1$ .
<b>11</b>	<b>2</b>	The addends are different according to the condition of the problem. Therefore 2 can be presented as $2 + 0$ . The greater of the two addends is 2.
<b>12</b>	<b>0</b>	I added 1 and got 2 as a result. Therefore the number that I added 1 to was 1. The result I should have gotten is $1 - 1 = 0$ .
<b>13</b>	<b>5</b>	Balloon + balloon + balloon = balloon + 10 cents. Therefore balloon + balloon = 10 cents. One balloon costs 5 cents.
<b>14</b>	<b>2</b>	One of the addends is $1 + 1 = 2$ , and the other is $2 - 2 = 0$ . The sum is $2 + 0 = 2$ .

<b>15</b>	<b>3</b>	I wrote down all numbers from 5 to 16: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16. The digits which have been used more than once are 1, 5 and 6. They are 3 in total.
<b>16</b>	<b>3</b>	John needs 3 balloons in order to have 4. John has 1 balloon. Peter has 1 balloon more than John. Peter has 2 balloons. Peter and John have $1 + 2 = 3$ balloons in total.
<b>17</b>	<b>6</b>	$\square = 1$ $\underbrace{\square}_1 + \Delta = 4 \Rightarrow \Delta = 3$ $\underbrace{\Delta}_3 + \odot = 9 \Rightarrow \odot = 6$
<b>18</b>	<b>6</b>	There were 6 flowers, each of which had 3 petals. I picked 4 of the flowers. Now there are 2 flowers in the vase with 3 petals each, i.e. the petals are 6.
<b>19</b>	<b>3</b>	A bunny eats 2 carrots every day. In two days it would eat 4 carrots. In three days it would eat 6 carrots.
<b>20</b>	<b>3</b>	$\square\Delta - 3 = 9 \Rightarrow \square\Delta = 12 \Rightarrow \square = 1, \Delta = 2 \Rightarrow \square + \Delta = 3$

**FINAL 2016: GROUP 1**

**Problem 1.**  $9 + 9 - 6 = 1\Box$   
 $\Box =$

- A) 2                                      B) 1                                      C) 0

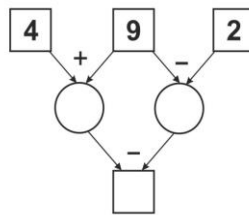
**Problem 2.** I have 17 roses – white, yellow and red. The white and yellow roses together are 10, the yellow and red roses together are also 10. The roses of which colour are the least in number?

- A) white                                      B) yellow                                      C) red

**Problem 4.** Adam, Bobby, Charles and Daniel won the top four places at a competition. Adam was ranked higher than Bobby, Charles was ranked lower than Daniel, and Bobby was ranked higher than Daniel. Who came third?

- A) Adam                                      B) Bobby                                      C) Daniel

**Problem 4.** Find the number  $\Box$  in the following diagram:



- A) 5                                      B) 6                                      C) 7

**Problem 5.** I will turn 15 in 8 years. How old was I 2 years ago?

- A) 5                                      B) 6                                      C) 7

**Problem 6.** How many of the numbers 0, 1, 2, 3, 4 and 5 can be places in the empty square, so that the following equation  $\Box + 5 < 9$  will be true?

- A) 3                                      B) 4                                      C) 5

**Problem 7.** What number will you get as a result of adding the numbers hidden under the shells?



- A) 10                                      B) 14                                      C) 16

**Problem 8.** You have 3 coins of 1 cent and 2 coins of 5 cents. How many different sums can be paid using three of these coins?





A) 3

B) 4

C) 5

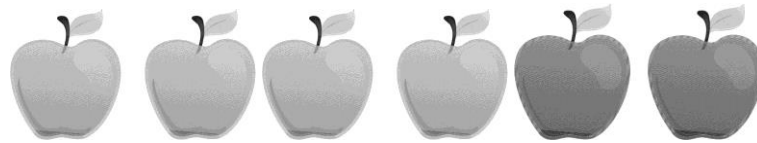
**Problem 9.** There are 3 teams participating in a football tournament. Each team played one game against each of the other teams. How many games have been played in total?

A) 3

B) 5

C) 6

**Problem 10.** There is a basket of apples in a dark room. Inside the basket there are 4 yellow and 2 red apples. What is the smallest number of apples we would need to take out (without looking), in order to be sure that we have taken out at least two yellow apples?



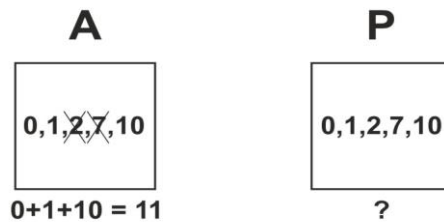
A) 3

B) 4

C) 5

**Problem 11.** The sum of two one-digit numbers is 17. The smaller number was subtracted from the greater number. What is the difference?

**Problem 12.** The numbers 0, 1, 2, 7 and 10 are written down on a piece of paper. Annika erased two of the digits and the numbers which remained on the piece of paper were 0, 1 and 10. When she added those numbers she got 11 as a sum. Pippi had her own piece of paper which also had the numbers 0, 1, 2, 7 and 10 written on it. She also erased two digits, correctly added the remaining numbers, but received a sum smaller than that of Annika. What is the smallest possible sum that Pippi could have gotten?



**Problem 13.** It is well known that when a die is rolled, the winning number is the one found on top of the die (1, 2, 3, 4, 5 or 6).



When the die shown on the picture was rolled, the winning number was 3. Three dice were rolled and there were three different winning numbers. The sum of the three numbers was 14. What is the smallest winning number we got?

**Problem 14.** Find the next (Fifth) sum:

First sum:

$$0 + 1 = 1$$

Second sum:

$$0 + 1 + 1 = 2$$

Third sum:

$$0 + 1 + 1 + 2 = 4$$

Fourth sum:

$$0 + 1 + 1 + 2 + 4 = 8$$

**Problem 15.** John arranged 12 books on his shelf. The book Pippi Longstocking was arranged 8th from left to right. Which place would the same book be at if we were counting from right to left?

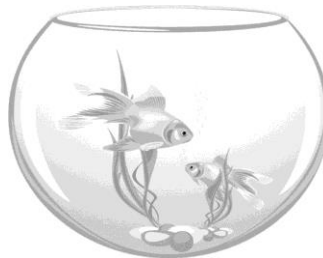
**Problem 16.** The number of one-digit numbers smaller than 5 is 1 less than the number of two-digit numbers smaller than  $\square$ . Find  $\square$ .

**Problem 17.** The numbers 1, 2, 3 and 4 must be placed in the following empty squares. Find the difference.

$$\begin{array}{c} \square + \square \\ \underbrace{\hspace{1.5cm}} \\ \square - \square \\ \underbrace{\hspace{1.5cm}} \end{array}$$

*difference*

**Problem 18.** Arnold and Mary have some pet fish. Mary has 2 fish more than Arnold. Together they have 18 fish. How many fish does Mary have?



**Problem 19.** All but 7 of a group of 18 children love ice cream. How many of the children do not love ice cream?

**Problem 20.** Peter and Steven had 18 sweets each. After eating a few of his, Peter has 11 sweets left, and Steven has eaten 10 sweets already. How many sweets are left in total?

## ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
<b>1</b>	<b>A</b>	$9 + 9 - 6 = 12 \Rightarrow \square = 2$
<b>2</b>	<b>B</b>	$\underbrace{\text{White} + \text{yellow}}_{10} + \text{red} = 17 \Rightarrow \text{red} = 7$ $\text{yellow} + \underbrace{\text{red}}_7 = 10 \Rightarrow \text{yellow} = 3 \Rightarrow \text{white} = 7$ <p>There are 3 yellow roses.</p>
<b>3</b>	<b>C</b>	<p>Adam was ranked higher than Bobby <math>\Rightarrow</math> the ranking is AB</p> <p>Bobby was ranked higher than Daniel <math>\Rightarrow</math> the ranking is ABD</p> <p>Charles was ranked lower than Daniel <math>\Rightarrow</math> the ranking is ABDC</p> <p>Daniel came third.</p>
<b>4</b>	<b>B</b>	$4 + 9 = 13; 9 - 2 = 7.$ Therefore $13 - 7 = 6 \Rightarrow \square = 6$
<b>5</b>	<b>A</b>	I will be 15 in 8 years. I am currently $15 - 8 = 7$ years old, and two years ago I was $7 - 2 = 5$ years old.
<b>6</b>	<b>B</b>	$0 + 5 < 9$ , correct; $1 + 5 < 9$ , correct; $2 + 5 < 9$ , correct; $3 + 5 < 9$ , correct; $4 + 5 < 9$ , wrong; $5 + 5 < 9$ , wrong. $\square + 5 < 9$ is correct for 4 of the numbers 0, 1, 2, 3, 4 and 5
<b>7</b>	<b>C</b>	The numbers are 1, 3, 5, 7, 9, 11 and 13. The numbers hidden under the shells are 5 and 11. Their sum is 16.
<b>8</b>	<b>A</b>	Three different sums can be paid using three of the coins: $1 + 1 + 1 = 3; 1 + 1 + 5 = 7; 1 + 5 + 5 = 11$
<b>9</b>	<b>A</b>	If the teams are A, B and C, then the games played were A and B, B and C, A and C. Three in total.
<b>10</b>	<b>B</b>	In the worst case scenario, we would first take out the two red apples. In this case the next two apples would definitely be yellow.
<b>11</b>	<b>1</b>	The two one-digit numbers are 8 and 9. Their difference is 1.

<b>12</b>	3	<p>On the piece of paper that has the numbers 0, 1, 2, 7 and 10 on it, Pippi can erase the following:</p> <p>10 - two digits 0 and 1; in which case the sum she would get is 10, which is smaller than that of Anika.</p> <p>7 and the first digit of 10; in which case the sum she would get is 3;</p> <p>7 and the second digit of 10; in which case the sum she would get is 4.</p> <p>2 and the first digit of 10; in which case the sum she would get is 8;</p> <p>2 and the second digit of 10; in which case the sum she would get is 9.</p> <p>1 and the first digit of 10; in which case the sum she would get is 9.</p>
<b>13</b>	3	<p>The possible options are: <math>14 = 6 + 6 + 2 = 6 + 5 + 3 = 6 + 4 + 4 = 5 + 5 + 4</math>. The winning numbers are different only in the second sum. The smallest winning number was 3.</p>
<b>14</b>	16	<p>The fifth sum is <math>0 + 1 + 1 + 2 + 4 + 8 = 16</math>.</p>
<b>15</b>	5	<p>After this book, there would be 4 other books if we count from left to right. If we count the books from right to left, the book would come after those 4 books, therefore it would come 5th.</p>
<b>16</b>	16	<p>The one-digit numbers smaller than 5 are 5: 0, 1, 2, 3, 4. The two-digit numbers smaller than 16 are 6: 10, 11, 12, 13, 14, 15.</p> <p>The number that we would need to place in the square is 16.</p>
<b>17</b>	2	<p><math>\frac{1+3}{4-2}</math> The difference is 2. difference</p>
<b>18</b>	10	<p><math>8 + 10 = 18</math>, therefore Mary has 10 fish.</p>
<b>19</b>	7	<p>Out of 18 children, all but 7 love ice cream. Therefore 7 of them do not love ice cream.</p>
<b>20</b>	19	<p>Peter has 11 sweets left and Steven has <math>18 - 10 = 8</math> sweets.</p> <p>There are <math>11 + 8 = 19</math> sweets left.</p>

## TEAM COMPETITION – NESSEBAR, BULGARIA

### MATHEMATICAL RELAY RACE

The answers to each problem are hidden behind the symbols @, #, &, § and \* and are used in solving the following problem. Each team, consisting of three students of the same age group, must solve the problems in 45 minutes and then fill a common answer sheet.

**Problem 1.** There are 14 chocolates in a box. Each of the three members of the team ate two chocolates. There are now @ chocolates left in the box. Find @.

**Problem 2.** There are # sparrows perched on a bush. @ of them flew off the bush. The remaining sparrows are 4 less than those who flew off. Find #.

**Problem 3.** I have # yellow and red flowers. Seven of them are tulips, and the rest are roses. Two of the flowers are yellow and the rest are red. What is the smallest possible number of red roses? Mark your answer by &. Find &.

**Problem 4.** Two identical chocolate bars cost as much as & identical sweets. Six chocolate bars cost as much as § sweets. Find §.

**Problem 5.** The two-digit numbers smaller than \* are §. Find \*.

### ANSWERS AND SHORT SOLUTIONS

Problem	Answer	Solution
1	@ = 8	$2+2+2=6$ , therefore 6 chocolates have been eaten already. There are $14-6=8$ chocolates left.
2	# = 12	The sparrows left on the bush are $8-4=4$ . At first the sparrows were $8+4=12$ .
3	& = 3	The roses are $12-7=5$ . In order to get the smallest possible number of red roses, both yellow flowers must be roses. $5-2=3$ , therefore at least 3 of the roses are red.
4	§ = 9	2 chocolate bars + 2 chocolate bars + 2 chocolate bars are equal to 3 sweets + 3 sweets + 3 sweets = 9 sweets.
5	* = 19	There are 9 numbers smaller than 19: 10, 11, 12, 13, 14, 15, 16, 17 and 18.